Sr. No. 753403

December 2023
M.Sc. (Mathematics) – IV SEMESTER
Fluid Dynamics (MATH21-853)

Time: 3 Hours **Max. Marks: 75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define viscosity. Also, give an example of a viscous fluid? (1.5)
(b) Explain Eulerian approach. (1.5)
(c) State Kelvin's theorem. (1.5)
(d) Define streak lines. (1.5)
(e) What do you mean by flows involving axial symmetry? (1.5)
(f) What are the images of a source in a rigid plane? (1.5)
(g) Define a doublet. (1.5)
(h) Define line source. (1.5)
(i) What do you mean by vortex rows? (1.5)
(j) State Milne Thompson circle theorem. (1.5)

PART -B

Q2 (a) Show that $\frac{x^2}{a^2b^2t^4} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form for the boundary of a liquid motion at any time t. (5)
(b) State and prove first potential theorem. (10)

Q3 Show that the fluid motion specified by $\vec{q} = \frac{k^2(x\hat{j} - y\hat{i})}{x^2+y^2}, (k = constant)$ is possible for an incompressible fluid flow and determine the equations of streamlines. Also test whether the motion is of potential kind and if so, determine the velocity potential. (15)

Q4 (a) Derive the equation of vorticity. (5)
(b) Prove that at any point P of a moving inviscid fluid, the pressure is the same in all directions, that is, p is independent of the orientation of δA. (10)

Q5 Doublets of strength μ_1 and μ_2 are situated at points A_1 and A_2 whose Cartesian coordinates are $(0,0,c_1), (0,0,c_2); c_1 > c_2$, their axis being directed towards and away from the origin respectively. Find the condition that there is no transport of fluid over the surface of the sphere $x^2 + y^2 + z^2 = c_1c_2$. (15)

Q6 (a) Derive the expression for stream function for a 2-D flow. (5)
(b) State and prove Weiss's Sphere theorem. (10)

Q7 Sate and prove theorem of Blasius. (15)