Sr. No 753404

JAN 2023

M.Sc.(Mathematics) - IV SEMESTER

Integral Equations (MATH21-854)

Time: 3 Hours

Max. Marks:75

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other*
4. *Notations used in the paper have their usual meanings.*

PART -A

Q1 (a) What is the Volterra Integral Equation of the second kind? (1.5)

(b) Find the iterated kernels for the following kernel (1.5)

$$K(x,t) = \cos(x - 2t), 0 \le x \le 2\pi, 0 \le t \le 2\pi.$$

(c) Find the Laplace transform of the function $F(t) = e^{at}$. (1.5)

(d) Define symmetric kernel $K(x,t)$. (1.5)

(e) Define Cauchy Integral. (1.5)

(f) What is the general form of the Abel Singular Integral Equation? (1.5)

(g) Write two applications of Green's function $G(x,t)$. (1.5)

(h) What is the Wronskian of the functions x, x^2 and x^3. (1.5)

(i) What is the solution of the integral equation $y(x) = 1 + \int_0^x y(u)du$. (1.5)

(j) State Hilbert formula. (1.5)

PART -B

Q2 (a) Solve the following integral equation by means of resolvent kernel (7)

$$y(x) = \sin x + 2\int_0^x e^{x-t} y(t)dt.$$

(b) Transform the following boundary value problem into an integral equation: (8)

$$\frac{d^2y}{dx^2} + xy = 1 \quad \text{with } y(0) = 0,\ y(l) = 1.$$

Q3 (a) Construct Green's function of the boundary value problem $y'' = 0, y(0) = y(1) = 0.$ (8)

(b) Find the solution of the integral equation $y(x) = \frac{5x}{6} + \lambda \int_0^1 xt\, y(t)dt.$ (7)

Q4 Convert the differential equation $y''(x) - \sin x\, y'(x) + e^x y(x) = x$ with initial conditions $y(0) = 1, y'(0) = -1$ to a Volterra integral equation of the second kind. Conversely, derive the original differential equation with initial conditions from the integral equation obtained. (15)

Q5 (a) Solve the Abel's equation $\int_0^t \frac{Y(x)}{(t-x)^{1/2}}dx = t(1+t)+1$ using Laplace Transform. (7)

(b) Show that the integral equation $y(x) = 1+\frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\, y(t)dt$ possesses infinitely many solutions. (8)

Q6 (a) Solve the integral equation (5)

$$f(x) = \int_x^b \frac{y(t)}{(\cos x-\cos t)^{\frac{1}{2}}}dt,\ 0 \leq a < x < b \leq \pi.$$

(b) Using Green's function, reduce the following boundary value problem to a Fredholm integral equation (10)

$$y'' + y = x,\ y(0) = y'(1) = 0.$$

Q7 Find the solution of the following Hilbert Singular integral equation of the second kind (15)

$ay(x) = f(x) - \frac{b}{2\pi}\int_0^{*2\pi} y(t)\cot\left(\frac{t-x}{2}\right)dt$, where a and b are complex constants.

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