Roll No. ....

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December 2023

B.Sc.(Physics) - I SEMESTER

Basic Calculus (OMTHP23-101)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define Continuity of a function and explain briefly different types of discontinuty. (1.5)

(b) Expand Sin(x+h) in infinite series expansion . Also show that (1.5)

$Sin(x+h) = sinxcosh + cosxsinh.$

(c) Evaluate $\lim_{x\to 0} \frac{x}{\sqrt{2+x} - \sqrt{2-x}}$ (1.5)

(d) At a given instant the major and minor axes of an ellipse are 60 cm and 40 cm respectively and they are increasing at the rate of 2 cm/sec and 1.5 cm/sec respectively. Find the rate at which the area is increasing at that instant. (1.5)

(e) Find the first order partial derivative of $u = cos^{-1}\left(\frac{x}{y}\right)$. (1.5)

(f) Find the nth derivative of $log\sqrt{\frac{2x+1}{x-2}}$. (1.5)

(g) Prove that $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive integer and m>1. (1.5)

(h) Evaluate $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\cos\theta\, dr\, d\theta$ (1.5)

(i) Change the order of integration $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)dydx$. (1.5)

(j) Prove that $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{15}} dx = 0$. (1.5)

PART –B

Q2 (a) Find all the asymptotes to the curve (8)

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$$

(b) Using Maclaurin's series , expand $\log(1+x)$.Hence deduce that (7)

$$log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \ldots \ldots \ldots$$

Q3 (a) By forming a differential equation, prove that (8)

$$cos(msin^{-1}x) = 1 - \frac{m^2}{2!}x^2 - \frac{m^2(2^2-m^2)}{4!}x^4 - \frac{m^2(2^2-m^2)(4^2-m^2)}{6!}x^6 \ldots \ldots$$

(b) If $u = sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial u^2}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{sinu\ cos2u}{4cos^3u}$ (7)

Q4 (a) If $x+y = 2e^{\theta}cos\varphi$ and $x-y = 2ie^{\theta}sin\varphi$, show that $\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial\varphi^2} = 4xy\frac{\partial^2 u}{\partial x\partial y}$. (8)

(b) If $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$, $w = \frac{z}{x-y}$, *show that* $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$. (7)

Q5 (a) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ meets the axes in A, B and C . Apply Dirichlet's integral to find the volume of the tetrahedron OABC. (8)

(b) Find , by double integration , the area lying inside the circle $r = a\ sin\theta$ and outside the cardioide $r = a(1-cos\theta)$. (7)

Q6 (a) Transform the following to Cartesian form and hence evaluate $\int_0^{\pi}\int_0^{a} r^3 sin\theta\, cos\theta dr\, d\theta$. (8)

(b) By using Transformation $x + y = u, y = uv$, show that $\int_0^1\int_0^{1-x} e^{\frac{y}{x+y}}\, dydx = \frac{1}{2}(e-1)$. (7)

Q7 (a) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. (8)

(b) Find the surface area of the solid generated by revolving the cycloid (7)

$x = a(\theta - sin\theta), y = a(1 - cos\theta)$ about the x-axis.
