6. (a) Show that

$$\Gamma(n)\int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy (n > 0) \qquad (5)$$

(b) Show that

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}. \qquad (5)$$

(c) Express the following integral in terms of gamma functions :

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} \qquad (5)$$

7. Solve the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-4)y = 0. \qquad (15)$$

321301

December 2023

B.Sc. (Physics) III SEMESTER

Mathematics Physics-II

(BPH–301A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) What are the Dirichlet's condition for any function to be expanded as fourier series. (1.5)

(b) Find a Fourier series to represent x^2 in the interval $(-l, l)$. (1.5)

(c) Show that : (1.5)

$$\frac{d}{dx}[x^{-n}]_n(x)] = -x^{-n}J_{n+1}(x)$$

(d) Show that : (1.5)

$$\Gamma(n+1) = n\Gamma(n)$$

(e) Evaluate : (1.5)

$$\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$$

(f) Find the roots of the indicial equation for the following differential equation (1.5)

$$9x(1-x)\frac{dy^2}{dx^2} - 12\frac{dy}{dx} + 4y = 0$$

(g) Show that

$$J_{½}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$$ (1.5)

(h) Write $f(x) = x^2 - 5x + 2$ in terms of Legendre polynomials. (1.5)

(i) Write the Parseval's Formula for the $f(x)$ converging uniformly in $(-l, l)$. (1.5)

(j) Evaluate : (1.5)

$$\beta\left(\frac{5}{2}, \frac{3}{2}\right)$$

PART–B

2. (a) Find the Fourier series expansion of $f(x) = 2x - x^2$ in $(0, 3)$. (10)

(b) Deduce that : (5)

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots - \infty = \frac{n}{12}$$

3. (a) Solve in series the following equation (10)

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$$

(b) Solve in series the following equation (5)

$$\frac{d^2y}{dx^2} + xy = 0$$

4. (a) Deduce the value of $J_{1/2}$. (5)

(b) Express $J_s(x)$ in terms of $J_0(x)$ and $J_1(x)$. (5)

(c) Show that (5)

$$\int_{-1}^{+1} x^2 P_{n-1} P_{n+1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

5. (a) Prove that (5)

$$\int_0^{\infty} e^{-x} L_m(x) L_n(x)\, dx = 0, m \neq n$$

(b) Show that (5)

$$H_{2n}(0) = (-1)^n \frac{2n!}{n!}$$

and, $H_{2n+1}(0) = 0$.

(c) Show that (5)

$$H_n(x) = 2nH_{n-1}(x)$$