Roll No. ....

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321101

December 2023

B.Sc. (Physics) - I SEMESTER

Mathematical Physics-I (BPH23-101T)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Solve the differential equation $sec^2x \tan y \, dx + sec^2y \tan x \, dy$. (1.5)

(b) Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$. (1.5)

(c) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point (2,–2,3). (1.5)

(d) Determine the value of δ_{ii} for i=1 to 3. (1.5)

(e) Prove that the solutions $y_1 = e^{-x}cosx$ and $y_2 = e^{-x}sinx$ of a differential equation are linearly independent. (1.5)

(f) What is permutation tensor? (1.5)

(g) Prove that $dx\, dy = r\, dr\, d\theta$ using Jacobian matrix. (1.5)

(h) Solve the differential equation $y \, logy \, dx + (x - logy)dy = 0$. (1.5)

(i) Find the volume of a tetrahedron whose vertices are the points A(2,–1,–3), B(4,1,3), C(3,2,–1) and D(1,4,2). (1.5)

(j) Determine the value of Div(r), where $r = \sqrt{x^2 + y^2 + z^2}$. (1.5)

PART -B

Q2 (a) Solve the differential equation $cot3x \frac{dy}{dx} - 3y = cos3x + sin3x, \; 0 < x < \frac{\pi}{2}$. (5)

(b) Solve the differential equation $y'' - 4y' + 4y = 8x^2e^{2x}sin2x$. (10)

Q3 (a) Find the value of λ, for which the differential equation $(xy^2 + \lambda x^2 y)dx + (x+y)x^2 dy = 0$ is exact. Also, solve the equation for this value of λ. (7)

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = sinhx + \sin\left((\sqrt{2})x\right)$. (8)

Q4 (a) Show that $div(grad\, r^n) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$. (7)

(b) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is solenoidal as well as irrotational. Find the scalar potential such that $\vec{F} = \vec{\nabla}\emptyset$. (8)

Q5 (a) Prove that $[(\vec{A} \times \vec{B}) \times \vec{C}] \times \vec{D} + [(\vec{B} \times \vec{A}) \times \vec{D}] \times \vec{C} + [(\vec{C} \times \vec{D}) \times \vec{A}] \times \vec{B} + [(\vec{D} \times \vec{C}) \times \vec{B}] \times \vec{A} = 0.$ (7)

(b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. (8)

Q6 (a) Evaluate $\iiint_V (2x + y)dV$, where V is closed region bounded by the cylinder $z=4-x^2$ and the planes x=0, y=0, y=2, and z=0. (5)

(b) Verify Stoke's theorem for the function $\vec{F} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$, where C is the unit circle in xy-plane bounding the hemisphere $x^2 + y^2 + z^2 = 1$. (10)

Q7 Verify the Gauss Divergence theorem for the function $\vec{F} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$. (15)
