Sr. No 321308

Dec 2023

B.Sc. Physics (Re)– III SEMESTER

Differential Equations (OMTH–301)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1 marks each) of Part –A in short.*
2. *Answer any three questions from Part –B in detail.*
3. *Different sub–parts of a question are to be attempted adjacent to each other.*
4. *Notations used in this paper have their usual meanings.*

PART –A

Q1 (a) Write the general form of the Exact differential equation. (1.5)

(b) Write down the standard form of the Lagrange's Linear equation. (1.5)

(c) Form the partial differential equation by eliminating the arbitrary constants from the equation $z = ax + by$. (1.5)

(d) What is the order of the differential equation $(\frac{d^2y}{dx^2})^2 + y = 0$. (1.5)

(e) What is the solution of the following partial differential equation (1.5)

$$\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + 9\frac{\partial^2 u}{\partial y^2} = 0.$$

(f) Find the general solution of the differential equation (1.5)

$(D^2 - 10)y = \sin x$, where $D \equiv \frac{d}{dx}$.

(g) What is the centre of the following power series (1.5)

$$\sum_{n=0}^{\infty} n!\,(x+7)^n.$$

(h) Define Power Series. (1.5)

(i) Solve the following differential equation: (1.5)

$$\frac{dy}{dx} - sin2x = ycotx .$$

(j) Solving by variation of parameter $y'' + y = xsinx$, the value of wronskian W is. (1.5)

PART –B

Q2 (a) Solve the following differential equation (8)

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2+1.$$

(b) Convert the 4th order ordinary differential equation $y''' + 3y'' - sint\,y' + 8y = t^2$ with initial conditions $y(0) = 1, y'(0) = 2, y''(0) = 3$ and $y'''(0) = 4$ to a system of four first order ordinary differential equations. (7)

Q3 (a) Find the general solution of the following differential equation (8)

$$x^2y'' + 4xy' + 2y = xlogx .$$

(b) Solve the following differential equation (7)

$$(y^3 - 2yx^2)dx + (2xy^2 - x^3)dy = 0.$$

Q4 Find the general solution in series of powers of x of the following differential equation: (15)

$$4xy'' + 2y' + y = 0.$$

Q5 (a) Find the general solution of the equation $\frac{y^2z}{x}p + xzq = y^2$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. (7)

(b) Solve the following differential equation by variation of parameter method: (8)

$$y'' + a^2 y = \sec ax'.$$

Q6 Show that the one–dimensional wave equation (15)

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0.$$

is hyperbolic, find an equivalent canonical form, and then obtain the general solution.

Q7 (a) Solve the following differential equation (8)

$$3\frac{dy}{dx} + y = e^{3x} y^4.$$

(b) Write down the polynomial $x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomial.

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