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Sr. No. 323306

December 2023 (Re-Appear)
B.Sc. (Hons.) Mathematics - III SEMESTER
Multivariate Calculus (BMH-303)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Find the domain and range of the function $f(x,y) = \sin xy$. (1.5)

(b) Define level curve. Also, give an example. (1.5)

(c) Find f_x as function if $f(x,y) = \frac{2y}{y+\cos x}$. (1.5)

(d) Define a vector field. (1.5)

(e) Define saddle point. Also, given an example. (1.5)

(f) Evaluate the integral $\int_0^2 \int_0^1 (x^2 + y^2)dx\, dy$. (1.5)

(g) Find the gradient field of the function $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$. (1.5)

(h) State fundamental theorem of line integrals. (1.5)

(i) Check whether the filed $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ is conservative or not? (1.5)

(j) State Gauss divergence theorem. (1.5)

PART -B

Q2 (a) Show that the function $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & for\ (x,y) \neq (0,0) \\ 0 & for\ (x,y) = (0,0) \end{cases}$ (5)

is continuous at every point except the origin.

(b) Find all the second-order partial derivatives of the function $f(x,y) = x\cos y + ye^x$. (5)

(c) Using the definition, find the derivative of $f(x,y) = 2x^2 + y^2$ of $P(-1,1)$ in the direction of the unit vector $\vec{u} = 3\hat{\imath} - 4\hat{\jmath}$. (5)

Q3 By using Lagrange's multiplier method, find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (15)

Q4 (a) Find the equations for the tangent plane and normal line at the point $P\ (1,-1,3)$ on the surface $x^2 + 2xy - y^2 + z^2 = 7$. (6)

(b) Find the directions in which $f(x,y) = x^2 + y^2$ (9)

(i) increases most rapidly at the point $(1,1)$.

(ii) Decreases most rapidly at the point $(1,1)$.

(iii) What are the directions of zero change in f at the point $(1,1)$.

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Q5 (a) Write an equivalent integral for $\int_0^2 \int_{x^2}^{2x} (4x+2)\,dy\,dx$ with the order of integration reversed by sketching the region of integration. Hence, evaluate the integral. (6)

(b) Evaluate $\int_0^3 \int_0^4 \int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \left(\frac{2x-y}{2}+\frac{z}{3}\right) dx\,dy\,dz$ by applying the transformation $u=\frac{2x-y}{2},\ v=\frac{y}{2},\ w=\frac{z}{3}.$ (9)

Q6 (a) Integrate $f(x,y,z) = x - 3y^2 + z$ over $C_1 \cup C_2$, where C_1 is line segment joining origin to $(1,1,0)$ and C_2 is line segment joining $(1,1,0)$ to $(1,1,1)$. (6)

(b) Using spherical polar coordinates (ρ, ϕ, θ), find the volume of region cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$. (9)

Q7 State both tangential and normal forms of Green's theorem. Verify both the forms for the field $\vec{F} = (x-y)\hat{\imath} + x\hat{\jmath}$ over the region R by the circle $C: x^2 + y^2 = 1$. (15)