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December 2023

B.Sc. (H) Mathematics- V SEMESTER

Discrete Mathematics (DEMH-501A)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *All the symbols used in this paper have their usual meanings.*

PART -A

Q1 (a) Prove that every subset of a countable set is countable. (1.5)

(b) Show that $n! \geq 2^{n-1}$, $n = 1,2,3,\ldots$ (1.5)

(c) Find the generating function for the numeric function $a_n = 10.2^n$. (1.5)

(d) Let $(L, \leq)$ be a lattice and $a, b, c \in L$. If $a \leq b \leq c$, then prove that (1.5)

$$a \vee b = b \wedge c.$$

(e) Define a Connected Graph. (1.5)

(f) Let $A = \{1,2,3,6\}$. Define a relation R on A as follows: (1.5)

$$x \, R \, y \; if \; x \; divides \; y$$

Draw the Hass diagram of the poset (A, R).

(g) Find the explicit formula for the finite sequence $87,82,77,72,67$. Can this sequence be described by a recursive relation? (1.5)

(h) Define validity of the argument. (1.5)

(i) Find the negation of the following proposition: (1.5)

$$-4 < x \leq 2.$$

(j) Express $x \wedge y$ in its complete sum-of-products form in three variables x, y, z. (1.5)

PART-B

Q2 (a) Prove that the following is true for any natural number n: (8)

$$1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1.$$

(b) Let $S = \{a, b, c\}$ and $\tilde{A}$=P(S) (power set of S). If the partial order on the set P(S) is set inclusion ($\subseteq$), then draw the Hasse diagram of the poset $(\tilde{A}, \subseteq)$. (7)

Q3 (a) Find the explicit formula for Fibonacci sequence using generating function method. (8)

(b) Find the solution of the difference equation $a_n = 2a_{n-1} - a_{n-2}$ with initial conditions $a_1 = 1.5$ and $a_2 = 3$. (7)

Q4 (a) Let $c = a * b$ (convolution of a and b) where $a = (a_0, a_1, \ldots, a_n, \ldots) = 2^n$ and $b = (b_0, b_1, \ldots, b_n, \ldots) = 4^n$ are the two numeric functions. Determine the generating function of c. (8)

(b) Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Then prove that (7)

$$a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c).$$

Q5 (a) If p, q *and* r are any propositions, then prove that $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$. (8)

(b) Check the validity of the following argument (7)

If you invest in the stock market, then you will get rich.
If you get rich, then you will be happy.
$\therefore$ If you invest in the stock market, then you will be happy.

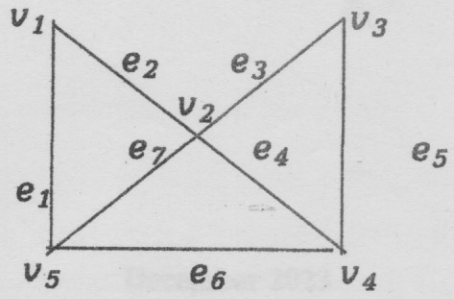
Q6 (a) Let n be a positive integer and D_n denotes the set of all divisors of n. Considering the partial order as divisibility on D_n, draw Hass diagram for D_{40}, D_{50} *and* D_{36} (8)

(b) In the following Graph, determine the following walks are path, simple paths, closed walks, circuits, simple circuits or are just walks: (7)

1. $v_5\, v_4\, v_2\, v_1$

2. $v_2\, v_3\, v_4\ \ v_5\, v_2$

3. $v_4\, v_2\, v_3\, v_4\ \ v_5\, v_2\, v_4$



Q7 (a) Prove that the five element lattices given in the following figures (Fig 1 and Fig 2) are non-distributive: (8)

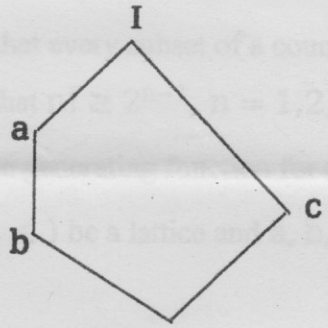


Fig:1

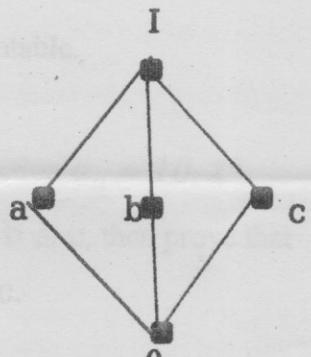


Fig:2

(b) Consider the Boolean polynomial $p(x, y, z) = (x \wedge y) \vee (x \vee (y' \wedge z))$. (7)

Construct the truth table for the Boolean function $f: B_3 \rightarrow B$ determined by this Boolean polynomial.