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Sr. No. 323301

December 2023

**B.Sc. (Hons.) MATHEMATICS - III SEMESTER**  
**Probability and Statistics (BMH-301A)**

Time: 3 Hours

Max. Marks:75

- Instructions:**
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
  2. Answer any four questions from Part -B in detail.
  3. Different sub-parts of a question are to be attempted adjacent to each other.
  4. Use of Scientific calculator is allowed.
  5. Use of **Normal Table** is allowed

**PART -A**

Q1 (a) Find the mean and variance of uniform probability distribution. (1.5)

(b) A continuous random variable X has the probability density function (1.5)

$$f(x) = \begin{cases} Kx & 0 \leq x < 2 \\ 2K & 2 \leq x < 4 \\ -Kx + 6K & 4 \leq x < 6 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of K.

(c) Three students A, B and C are running in a race. A and B have the same (1.5)  
probability of winning and each is twice as likely to win as C. Find the  
probability that B or C wins.

(d) If the sum of the mean and variance of binomial distribution of 5 trails (1.5)  
is 4.8, find the distribution.

(e) Write any three properties of Normal probability distribution. (1.5)

(f) What do you understand by Poisson probability distribution? Explain. (1.5)

(g) Explain the following: (1.5)

1. Joint probability distribution function
2. Joint continuous density function
3. Joint moment generating function

(h) The joint probability density function of (X, Y) is given by (1.5)

$$f(x,y) = \begin{cases} 2 & ; 0 < x < 1, 0 < y < 1 \\ 0 & ; \text{elsewhere} \end{cases}$$

Find the marginal density function of X and Y.

(i) Explain the merits and demerits of Karl Pearson's Coefficient of (1.5)  
Correlation.

(j) Write any three properties of Regression Coefficient. (1.5)

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**PART -B**

- Q2 (a) Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings. (4)

- (b) If the probability density function of a random variable is given by (4)

$$f(x) = \begin{cases} K(1-x^2) & , \quad 0 \leq x < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find the mean and variance.

- (c) Find the moment generating function of a random variable X having the probability density function (7)

1.  $f(x) = \begin{cases} 1/3 & , \quad -1 < x < 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$

2.  $f(x) = \begin{cases} x & , \quad 0 \leq x < 1 \\ 2-x & , \quad 1 \leq x < 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$

- Q3 (a) Find the mean, variance and mode of Binomial probability distribution. (8)

- (b) The customer accounts of certain department store have an average balance of Rs. 120 and Standard deviation of Rs. 40. Assuming that the account balances are normally distributed. (7)

1. What proportion of the account is over Rs. 150 ?
2. What proportion of account is between Rs. 100 and Rs. 150 ?
3. What proportion of account is between Rs. 60 and Rs. 90 ?

- Q4 (a) Three coins are tossed. Let X denotes the number of heads on first two coins and Y denote the number of heads on last two coins. Find (7)

1. Joint probability distribution of X and Y.
2.  $E[Y / X = 1]$
3.  $Cov(X, Y)$
4.  $\rho(X, Y)$

- (b) The joint probability density function of two dimensional random variable (X, Y) is given below: (8)

$$f(x,y) = \begin{cases} \frac{8}{9}xy & ; \quad 1 \leq x \leq y < 2 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

1. Find the marginal density function of X and Y.
2. Find the conditional density function of Y given  $X = x$  and conditional density function of X given  $Y = y$ .
3.  $Cov(X, Y)$
4.  $\rho(X, Y)$

- Q5 (a) The following table gives the number of students having different heights and weights: (8)

Height in Centimeters	Weight in Kilograms					Total
	55-60	60-65	65-70	70-75	75-80	
150-155	1	3	7	5	2	18
155-160	2	4	10	7	4	27
160-165	1	5	12	10	7	35
165-170	---	3	8	6	3	20
<b>Total</b>	<b>4</b>	<b>15</b>	<b>37</b>	<b>28</b>	<b>16</b>	<b>100</b>

Find the coefficient of correlation between the heights and weights of the students.

- (b) From the given data obtain the two regression equations using the method of least square (7)

X	1	2	3	4	5	6	7	8
Y	3	7	10	12	14	17	20	24

- Q6 (a) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards drawn and found to be hearts. Find the probability of the missing card to be a heart. (6)

- (b) If X and Y are discrete random variables, then prove that: (2)

$$E(X + Y) = E(X) + E(Y)$$

- (c) Fit a Poisson distribution of the following data and calculate theoretical frequencies. (7)

Death	0	1	2	3	4
Frequency	122	60	15	2	1

- Q7 (a) If X and Y are two random variables having joint density function: (4)

$$f(x, y) = \begin{cases} \frac{1}{8} (6 - x - y) & ; 0 < x < 2, \quad 2 < y < 4 \\ 0 & ; \text{elsewhere} \end{cases}$$

Find

- $P[X < 1 \cap Y < 3]$
- $P[Y < 3 / X < 1]$

- (b) Consider a sample space of size 2 drawn without replacement from an urn containing three balls, numbered 1, 2 and 3. Let X be the number on the first ball drawn and Y the larger of the two numbers drawn. (3)
- Find the joint discrete density function of X and Y.
  - $Cov(X, Y)$

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- (c) Calculate the Karl Pearson's coefficient of correlation from the following data: (6)

X	24	27	28	28	29	30	32	33	35	40
Y	18	20	22	25	22	28	30	27	30	22

- (d) From the data given below, calculate the coefficient of rank correlation between X and Y: (2)

X	78	89	97	69	59	79	68	57
Y	125	137	156	112	107	136	123	108

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