(b) An elastic string of natural length 2a can just support a certain weight when it is stretched till its whole length is 3a. One end of the string is now attached to a point on a smooth horizontal table and the same weight is attached to the other end. Prove that if the weight be pulled to any distance and let go, the string will become slack after a time $\frac{1}{2}\pi\sqrt{\frac{a}{g}}$. (9)

7. Two smooth inclined planes of equal heights and inclinations α and β (α > β) are places back-to-back. Masses m_1 and m_2 ($m_1 > m_2$) resting on them are connected by a light inextensible and flexible string passing over a smooth, fixed, small, light pulley at the common vertex of the two planes. If the system is free to move, find acceleration of the masses and the tension in the string. Also, find the mathematical condition for the system to move with uniform velocity if it sets into motion. (15)

December 2023

B.Sc. (Hons.) Mathematics – V SEMESTER

Mechanics–I (BMH–501A)

Time : 3 Hours]   [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*

2. *Answer any four questions from Part-B in detail.*

3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Resolve a force of magnitude 100 N into two components making an angle 60° and 90° with it on the opposite sides. (1.5)

   (b) State Lami's Theorem. (1.5)

   (c) State Varignon's theorem. (1.5)

   (d) Show that the direction of resultant of two consecutive forces always inclines more towards the greater force. (1.5)

   (e) Two unlike parallel forces 8 and 12 Newtons act at two points on a body, which are 1 meter apart. Find their resultant. (1.5)

(f) What does the slope of the curve in velocity-time graph of the particle at time t represents? Also, justify your answer. (1.5)

(g) Define uniform acceleration. (1.5)

(h) When a lift, carrying a mass m, is moving down with a uniform velocity, then write the mathematical expression of the pressure on the lift. (1.5)

(i) What do you mean by a periodic motion? (1.5)

(j) State Hooke's Law. (1.5)

PART–B

2. (a) Two forces of magnitude $P + Q$ and $P - Q$ make an angle 2α with one another, and their resultant makes an angle θ with the bisector of the angle between them. Show that $P \tan \theta = Q \tan \alpha$. (6)

(b) Using the triangle law of forces, find a point O within a quadrilateral ABCD such that if a particle placed at it be acted upon by the forces represented by OA, OB, OC and OD respectively, it will be in equilibrium. (9)

3. (a) A man on the ground is pulling a vertical tree with a given force by means of a rope p meters long. At what point of the tree must one end of the rope be attached so that he may have the greatest tendency to pull it over? (6)

(b) Find the resultant of two parallel forces acting upon a rigid body. (9)

4. (a) A force of 10 kg is acting at the point (5, 0) of xy-plane making an angle 30^0 with the positive direction of x-axis. A couple of moment 20 kg . m in the clockwise direction is added to it. What is the effect of this couple on the given force? At what point will the final resultant meet x-axis? (6)

(b) Six coplanar forces act on a rigid body along the sides AB, BC, CD, DE, EF and FA of a regular hexagon of side 1 unit. Their magnitudes are 10, 20, 30, 40, P and Q units respectively. Find P and Q so that the system reduces to a couple. Also, find the moment of this couple. (9)

5. (a) A bus is beginning to move with an acceleration of $\frac{1}{3}$ m/s^2. A man who is $13\frac{1}{3}$ m behind the bus starts running at 3 m/s to catch the bus. After what time will the man be able to catch the bus? Explain the double answer. Find the time if the man were $13\frac{1}{2}$ m behind. (6)

(b) In rectilinear motion with uniform acceleration, find out the general expression of the distance travelled in the nth second. Use it to solve the problem: A body falls freely from the top of a tower and during the last second of its flight it falls $\frac{16}{25}$ th of the whole distance. What is the height of the tower? (9)

6. (a) A particle moving with S.H.M. of period 12 seconds, travel 10 cm from the position of rest in 2 seconds. Find the amplitude, the maximum velocity and the velocity at the end of 2 seconds. (6)