11/12 (30)

Sr. No 323507

Dec-2023(Reappear)

B.Sc.(H)(Mathematics) V SEMESTER

Special Functions and Integral Transform(DEMH -503)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Que.1(a) Define radius of convergence of a power series.

(b) State generating function for Bessel's equation.

(c)Define ordinary and singular points of a differential equation.

(d)Briefly explain Hermite polynomial for even and odd values of 'n'.

(e) State orthogonality relation of Legendre's polynomial.

(f)Find the Laplace Transform of $e^{4t} + e^{2t} + t^3 + \sin^2 t$.

(g)State and Prove the first shifting theorem of Laplace Transform.

(h)State the convolution theorem of Laplace Transform.

(i)State and Prove the Linearity property of Fourier Transform.

(j) State and prove modulation property for Fourier transforms. (1.5*10 = 15)

PART-B

Que.2(a) Find the power series solution of the given differential equation (8)

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (3x+2)y$ in powers of x, (i.e., about x = 0).

(b) Show that $J_{-n}(x) = (-1)^n J_n(x)$, where n is a positive integer. (7)

Que.3(a)Prove that $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$. (8)

(b)Express $H(x) = x^4+2x^3+2x^2 - x - 3$ in terms of Hermitian polynomial. (7)

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Que.4(a) Find the Laplace transform of the function $f(t) = \begin{cases} t, & 0 \le t < 2 \\ 3, & t > 2 \end{cases}$. (8)

(b) Using convolution theorem, find the inverse Laplace Transform of the function $\frac{s}{(s^2+a^2)^3}$. (7)

Que.5(a) Find the Fourier transform of the function $f(x) = \begin{cases} k, & 0 < x < a \\ 0, & otherwise \end{cases}$. (8)

(b) State and Prove the relation between Fourier Transform and Laplace transform. (7)

Que.6(a) Find the solution of $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4\left(x^2 - \frac{n^2}{x^2}\right)y = 0$ in terms of Bessel's function. (8)

(b)Show that $(2n+1)(1-x^2)\,P_n'(x) = n(n+1)[P_{n-1}(x) - P_{n+1}(x)]$. (7)

Que.7(a) Using Laplace transform ,evaluate $t^2\cos at$. (8)

(b)State and Prove the Parseval's identity for Fourier Transform. (7)