Sr. No. 323506

December 2023

B.Sc.(H)MATHEMATICS (Re-Appear) - V SEMESTER

DISCRETE MATHEMATICS (DEMH-501)

o/c

Time: 3 Hours | Max. Marks:75

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Explain the following terms: (1.5)

1. Countable sets
2. Antisymmetric relation
3. Chain

(b) Explain Well Ordered Set with example. (1.5)

(c) Find the recursive formula for the factorial function. (1.5)

(d) Find the generating function of the numeric function (1.5)

$$a_n = 5 \cdot 2^n, \quad n \geq 0$$

(e) Differentiate between Conjunction, Disjunction and Negation. (1.5)

(f) Explain Isomorphic Ordered Sets with example. (1.5)

(g) Construct the Truth table for the following statement (1.5)

$$p \wedge (p \vee q)$$

(h) Define the following terms: (1.5)

1. Quantifiers
2. Universal Statement
3. Existential Statement

(i) Differentiate between Lattice and Dual Lattice. (1.5)

(j) Show that a Lattice with three or fewer elements is a chain. (1.5)

PART –B

Q2 (a) Explain Partially ordered set with example and hence show that, if R is a partial order on a set A then R^{-1} (inverse relation) is also a partial order. (8)

(b) Let n be a positive integer and D_n denote the set of all divisors of n. Considering the partial order of divisibility in D_n, draw Hasse diagram of D_{24}, D_{30}, D_{36} and D_{18}. (7)

Q3 (a) Find the particular solution of the difference equation (8)

$$a_n - a_{n-1} - 2a_{n-2} = 2n^2$$

Also, write down the total solution.

(b) Show that the generating function (7)

$$\frac{1}{(1-4z)^2}$$

Can be expressed as

$$a_n = (n+1)4^n$$

Q4 (a) Construct the Truth table for the following statements: (8)

1. $p \oplus q$

2. $(p \wedge q) \vee \sim r$

(b) Verify the validity of the following argument form: (7)

$$p \rightarrow q$$
$$p$$
$$\therefore q.$$

Q5 (a) Let $(L, \leq)$ be a lattice and if $a, b \text{ and } c \in L$ then show that, (8)

$$a \leq c \text{ if and only if } a \vee (b \wedge c) \leq (a \vee b) \wedge c$$

(b) Prove that, the direct product of any distributive lattices is a distributive lattice. (7)

Q6 (a) Use mathematical induction to show that (8)

$$n! \geq 2^{n-1}, \quad n = 1, 2, 3, - - - -$$

(b) Find an explicit formula for the sequence defined by the recurrence relation (7)

$$a_n = a_{n-1} + 2a_{n-2}, \quad n \geq 2$$

With the initial conditions

$$a_0 = 1 \quad and \quad a_1 = 8$$

Q7 (a) Show that the following argument is invalid. (8)

If taxes are lowered, then income rises

income rises

$\therefore$ *Taxes are lowered*

(b) Let $(L, \leq)$ be a lattice and if $a, b \text{ and } c \in L$, Then (7)

1. $a \vee (b \vee c) = (a \vee b) \vee c$

2. $(a \wedge b) = (b \wedge a)$

3. $a \wedge (a \vee b) = a$