

December 2023

B.Sc. (Mathematics) and B.sc (Mathematics & Computing) -(Re-appear)

I SEMESTER

olc

Algebra (BMH-102A)

Time: 3 Hours

Max. Marks:75

- Instructions:
- 1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
- 2. Answer any four questions from Part -B in detail.
- 3. Different sub-parts of a question are to be attempted adjacent to each other.
- 4. Any other specific instructions

PART -A

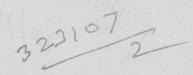
- Q1 (a) Find rank of the matrix $\begin{bmatrix} 0 & 6 & 6 & 1 \\ -8 & 7 & 2 & 3 \\ -2 & 3 & 0 & 1 \\ -3 & 2 & 1 & 1 \end{bmatrix}$ (1.5)
 - (b) If A is non zero column matrix and B is non zero row matrix, show that rank of (AB) is 1. (1.5)
 - (c) Examine the linear dependence and linear independence of the following set of vectors (1.5) (1,2,3),(3,-2,1),(1,-6,-5).
 - (d) Find matrix of the bilinear form (1.5)

$$10x_1y_1 + 9x_1y_2 - 6x_2y_1 - 2x_2y_2 + 14x_2y_3 - 14x_3y_2 + 30x_3y_3.$$

- (e) Transform the equation $x^5 + 8x^4 9x^3 + 13x^2 12x + 9 = 0$ into another whose roots (1.5) shall be equal in magnitude but opposite in sign to the roots of the given equation.
- (f) Find the condition that roots of the equation $x^3 + px^2 + qx + r = 0$ are such that the (1.5) product of two roots is -1.
- (g) Solve the equation $32x^3 48x^2 + 22x 3 = 0$, roots being in A.P. (1.5)
- (h) Remove the fractional coefficients from the equation $x^4 + \frac{7}{2}x^3 \frac{9}{5}x^2 \frac{1}{3}x + 10 = 0.$ (1.5)
- (i) Find the remainder when $x^3 + 3px + q$ be divided by $(x a)^2$. (1.5)
- (j) Find the equation of lowest degree with rational coefficients which has $4.2 + \sqrt{3}$ as its (1.5) roots..

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PART-B



- Q2 (a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. (8)
 - (b) Show that if A is Hermitian and P is unitary, then $P^{-1}AP$ is Hermitian (7)
- Q3 (a) Diagonalize the quadratic form $-21x^2 11y^2 2z^2 + 30xy 12xz + 8yz$. Show that it is negative semi-definite. (8)
 - (b) If A is real and skew-symmetric matrix such that $A^2 + I = 0$, then show that A is orthogonal and is of even order. (7)
- Q4 (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & 2 \end{bmatrix}$. Also find the minimal polynomial of the matrix.
 - (b) Solve the system of homogeneous equations

$$x + 3y - 2z = 0; 2x - y + 4z = 0; x - 11y + 14z = 0$$
(7)

- Q5 (a) Find the quotient and remainder using synthetic division, when $3x^4 + 2x^2 + x + 1 \text{ is divided by } 2x + 1.$
 - (b) Show that the same transformation can remove both second and fourth terms of the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ and hence solve it.
- Q6 (a) Solve the equation if sum of two roots is equal to sum of other two roots $x^4 8x^3 + 21x^2 20x + 5 = 0.$ (8)
 - (b) Solve the equation $40x^4 22x^3 21x^2 + 2x + 1 = 0$ whose roots are in H.P. (7)
- Q7 (a) Solve the equation $x^4 15x^2 10x + 24 = 0$ by Descarte's Method. (8)
 - (b) If α, β, γ are the roots of cubic $x^3 + 3x + 2 = 0$ find the equation whose roots are $(\alpha \beta)(\alpha \gamma), (\beta \gamma)(\beta \alpha), (\gamma \alpha)(\gamma \beta). \tag{7}$
