Sr. No 323107

December 2023

B.Sc. (Mathematics) and B.sc (Mathematics & Computing) –(Re-appear)

I SEMESTER

Algebra (BMH-102A)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. ~~*Any other specific instructions*~~

PART –A

Q1 (a) Find rank of the matrix $\begin{bmatrix} 0 & 6 & 6 & 1 \\ -8 & 7 & 2 & 3 \\ -2 & 3 & 0 & 1 \\ -3 & 2 & 1 & 1 \end{bmatrix}$ (1.5)

(b) If A is non zero column matrix and B is non zero row matrix , show that rank of (AB) is 1. (1.5)

(c) Examine the linear dependence and linear independence of the following set of vectors (1.5) (1,2,3),(3,-2,1),(1,-6,-5).

(d) Find matrix of the bilinear form (1.5)

$$10x_1y_1 + 9x_1y_2 - 6x_2y_1 - 2x_2y_2 + 14x_2y_3 - 14x_3y_2 + 30x_3y_3.$$

(e) Transform the equation $x^5 + 8x^4 - 9x^3 + 13x^2 - 12x + 9 = 0$ into another whose roots (1.5) shall be equal in magnitude but opposite in sign to the roots of the given equation.

(f) Find the condition that roots of the equation $x^3 + px^2 + qx + r = 0$ are such that the (1.5) product of two roots is -1.

(g) Solve the equation $32x^3 - 48x^2 + 22x - 3 = 0$, roots being in A.P. (1.5)

(h) Remove the fractional coefficients from the equation $x^4 + \frac{7}{2}x^3 - \frac{9}{5}x^2 - \frac{1}{3}x + 10 = 0$. (1.5)

(i) Find the remainder when $x^3 + 3px + q$ be divided by $(x - a)^2$. (1.5)

(j) Find the equation of lowest degree with rational coefficients which has $4,2 + \sqrt{3}$ as its (1.5) roots..

PART –B

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PART –B

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Q2 (a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. (8)

(b) Show that if A is Hermitian and P is unitary, then $P^{-1}AP$ is Hermitian (7)

Q3 (a) Diagonalize the quadratic form $-21x^2 - 11y^2 - 2z^2 + 30xy - 12xz + 8yz$. Show that it is negative semi-definite. (8)

(b) If A is real and skew- symmetric matrix such that $A^2 + I = O$, then show that A is orthogonal and is of even order. (7)

Q4 (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & 2 \end{bmatrix}$. Also find the minimal polynomial of the matrix. (8)

(b) Solve the system of homogeneous equations (7)

$$x + 3y - 2z = 0; 2x - y + 4z = 0; x - 11y + 14z = 0$$

Q5 (a) Find the quotient and remainder using synthetic division , when (8)

$3x^4 + 2x^2 + x + 1$ is divided by 2x+1.

(b) Show that the same transformation can remove both second and fourth terms of the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ and hence solve it. (7)

Q6 (a) Solve the equation if sum of two roots is equal to sum of other two roots (8)

$x^4 - 8x^3 + 21x^2 - 20x + 5 = 0$.

(b) Solve the equation $40x^4 - 22x^3 - 21x^2 + 2x + 1 = 0$ whose roots are in H.P. (7)

Q7 (a) Solve the equation $x^4 - 15x^2 - 10x + 24 = 0$ by Descarte's Method. (8)

(b) If α, β, γ are the roots of cubic $x^3 + 3x + 2 = 0$ find the equation whose roots are (7)

$(\alpha - \beta)(\alpha - \gamma), (\beta - \gamma)(\beta - \alpha), (\gamma - \alpha)(\gamma - \beta)$.

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