Roll No. ....   Total Pages : 3

323101

December 2023

B.Sc. (Hons.) Mathematics/B.Sc. (Mathematics & Computing) – I SEMESTER

Calculus–I (BMH23–101)

Time : 3 Hours]   [Maximum Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any **four** questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Check the continuity of the function $f(x) = \frac{x^2 + 2x - 3}{x - 1}$ at $x = 1$. (1.5)

   (b) Find $\frac{dy}{dx}$ if $y = \cos(x^3)$. (1.5)

   (c) Give an example of a non-differentiable function. (1.5)

   (d) Find $f^n(x)$ if $f(x) = 3x^4 - 2x^3 + x^2 - 4x + 2$. (1.5)

(e) State Rolle's theorem. Also, give its geometric interpretation. (1.5)

(f) Find $\frac{ds}{dt}$ if $s = (1+t)\sqrt{t}$. (1.5)

(g) Change the polar coordinates $\left(-3, \frac{5\pi}{4}\right)$ to Cartesian coordinates. (1.5)

(h) Sketch the graph of $\theta = \frac{\pi}{6}$ in polar coordinates. (1.5)

(i) Find the critical points of $f(x) = (x-1)^2(x+2)$. (1.5)

(j) Find where the graph of $f(x) = x^3 + 3x + 1$ is concave up and where it is concave down. (1.5)

PART–B

2. (a) Prove that $\lim_{x \to +\infty} \frac{1}{x} = 0$. (7)

(b) Find:

(i) $\lim_{x \to +\infty} x^{\frac{1}{x}}$.

(ii) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$. (8)

3. (a) Prove that if a function f is differentiable at c, then f is continuous at c. Also, give an example of a differentiable function. (7)

(b) State and prove Leibnitz's theorem. (8)

4. (a) State and prove Cauchy's mean value theorem. (7)

(b) Find the Maclaurin's series expansion of $f(t) = e^t$ using Lagrange's form of remainder. (8)

5. (a) Sketch the graph of the equation $y = x^3 - 3x + 2$ and identify the location of the intercepts, relative extrema and inflection points. (7)

(b) Sketch the graph of the equation $r = a(1 - \cos\theta)$ in polar coordinates, assuming a to be a positive constant. (8)

6. (a) Let f be a differentiable function on the closed interval [a, b] of finite length and suppose $f'(a) < y_0 < f'(b)$. Prove that there is a point x_0 in the open interval (a, b) such that $f'(x_0) = y_0$. (7)

(b) Sketch the graph of the function

$$f(x) = \frac{3x-5}{x-2}.$$ (8)

7. (a) Find the n^{th} derivative of x^2e^{3x}. (7)

(b) Prove that if limit of a function at a point exists, then it is unique. (8)