(b) By using spherical polar coordinates and triple integral, evaluate the volume of the cone $z = \sqrt{x^2 + y^2}$. (9)

7. (a) Find the outward flux of $\vec{F} = yz\hat{i} + x\hat{j} - z^2\hat{k}$ through the parabolic cylinder $y = x^2$, $0 \le x \le 1$, $0 \le z \le 4$. (6)

(b) Verify Gauss Divergence theorem for the vector filed $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 - a^2$. (9)

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323303

December 2023

B.Sc(Maths/MAC) III SEMESTER

Multivariate Calculus (BMH-303A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find the domain and range of the function $f(x, y) = \sin^{-1}(y - x)$. (1.5)

(b) Find an equation for the level surface of the function $f(x, y, z) = \sqrt{x - y} - \log z$ through the point (3, –1, 1). (1.5)

(c) Give an example of a function which is not continuous at a point but possesses both its first order partial derivatives at that point. (1.5)

(d) Find the critical points for the function $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$. (1.5)

(e) Find the equation normal line to the surface $x^2 + y^2 + z = 9$ at the point (1,2,4). (1.5)

(f) Evaluate the integral $\int_0^{2\pi}\int_0^{\pi/3}\int_0^1 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

(g) State Fubini's second theorem. (1.5)

(h) Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C' joining origin to the point (1,1,1). (1.5)

(i) Define a conservative vector field. (1.5)

(j) State Green's theorem. (1.5)

PART–B

2. (a) Show that the function $f(x,y) = \dfrac{x^2 + y}{y}$ has no limit as $(x, y) \to (0, 0)$. (6)

(b) State and prove chain rule for a function of two independent variables. Verify this rule for the function $w = xy, x = \cos t, y = \sin t$ at $t = \dfrac{\pi}{2}$. (9)

3. (a) Find all the Second-order partial derivatives of the function $f(x, y) = \tan^{-1}\left(\dfrac{y}{x}\right)$. (6)

(b) If ς is a smooth curve on the level surface $f(x, y, z) = c$ of a differentiable function f, then prove that at every point along the curve, ∇f is orthogonal to the curve's velocity vector. Use this result to find the parametric equations of the tangent line to the curve of intersection of the surfaces $x^2 + y^2 - 2 = 0$ and $x + z - 4 = 0$ at the point P (1, 1, 3). (9)

4. (a) Find the absolute maxima and minima of $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant. (6)

(b) The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. By using Lagrange's multiplier method, find the points on the ellipse that lie closest to and farthest from the origin. (9)

5. (a) By using double integrals, find the area of the cardioid $r = 1 + \cos\theta$. (6)

(b) By changing the order of integration of $\int_0^\infty\int_0^\infty e^{-xy} \sin px \, dx dy$, show that $\int_0^\infty \dfrac{\sin px}{x} dx = \dfrac{\pi}{2}$. (9)

6. (a) A fluid's velocity field is $\vec{F} = x\hat{i} + z\hat{j} + y\hat{k}$. Find the flow along the helix $\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j} + t\hat{k}, 0 \le t \le \dfrac{\pi}{2}$. (6)