

December 2023

B.Sc.(Life Science) I SEMESTER

Basic Mathematics (OMTHO-23-101)

Time: 3 Hours

Max. Marks:75

- Instructions:**
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
  2. Answer any four questions from Part -B in detail.
  3. Different sub-parts of a question are to be attempted adjacent to each other.
  4. Any other specific instructions

**PART -A**

- Q1 (a) Find the limit of  $\lim_{x \rightarrow 1} \frac{x^2+x-2}{x^2-1}$ . (1.5)
- (b) Show that the function  $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$  is continuous at  $x=0$ . (1.5)
- (c) Find the derivative of  $\sec(\tan^{-1}x)$ . (1.5)
- (d) Find the derivative of  $e^x(x^2 + \log x)$ . (1.5)
- (e) Find two positive numbers whose product is 49 and the sum is minimum. (1.5)
- (f) Write the formula of cross product of three vectors. (1.5)
- (g) If the two vectors  $\vec{a}$  and  $\vec{b}$  are such that  $|\vec{a}| = 2$ , and  $|\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 4$  then find the value of  $|\vec{a} - 2\vec{b}|$ . (1.5)
- (h) Evaluate  $\int \sqrt{1-x^2} dx$ . (1.5)
- (i) Evaluate  $\int_1^2 \frac{e^{1/x}}{x^2} dx$ . (1.5)
- (j) Find  $|\vec{x}|$  if  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$ , where  $\vec{a}$  is a unit vector. (1.5)

**PART -B**

Q2 (a) Find from first principle , the derivative of  $\cos(x^2+1)$ . (5)

(b) Show that the limit  $\lim_{x \rightarrow 0} \left( \frac{e^{1/x} - 1}{e^{1/x} + 1} \right)$  does not exist. (5)

(c) Differentiate  $e^{\sqrt{\cot x}}$ . (5)

Q3 (a) If  $y = \frac{1-\tan x}{1+\tan x}$ , show that  $\frac{dy}{dx} = \frac{-2}{(1+\sin 2x)}$ . (8)

(b) In a certain culture of bacteria , the rate of increase of bacteria is proportional to the number of bacteria present. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours . Determine the number of bacteria present in the beginning. (7)

Q4 (a) Differentiate  $\tan^{-1} \left\{ \frac{x^{1/3} + a^{1/3}}{1 - x^{1/3} a^{1/3}} \right\}$ . (8)

(b) If  $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$ , find  $\frac{dy}{dx}$ . (7)

Q5 (a) Evaluate  $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$ . (8)

(b) Evaluate  $\int \frac{2x^2+1}{x^2-3x+2} dx$ . (7)

Q6 (a) If  $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$  and  $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$  then find the value of  $\lambda$  so that the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are orthogonal. (8)

(b) Find the angle between unit vectors  $\vec{a}$  and  $\vec{b}$  so that  $(\sqrt{3}\vec{a} - \vec{b})$  is also a unit vector. (7)

Q7 (a) Evaluate  $\int_0^{\pi/2} \frac{\sin x}{\sqrt{1+\cos x}} dx$ . (8)

(b) Differentiate  $\sin^{-1} \left( \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right)$ . (7)

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