

December 2023

B.Tech. (Robotics and Artificial Intelligence) - V SEMESTER
Probability and Statistics (BSC-RAI-501-21)

Time: 3 Hours

Max. Marks:75

- Instructions:**
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
 2. Answer any four questions from Part -B in detail.
 3. Different sub-parts of a question are to be attempted adjacent to each other.
 4. Use of calculator is allowed.
 5. Use of Normal table is allowed.

PART -A

- Q1 (a) The probability density function (1.5)

$$f(x) = \begin{cases} kx^3, & 0 \leq x \leq 3 \\ 0, & \text{Otherwise} \end{cases}$$

Find the value of k and the probability between $x = \frac{1}{2}$ and $x = \frac{3}{2}$

- (b) If the sum of the mean and the variance of Binomial distribution of five trails is 4.8, find the distribution. (1.5)

- (c) Calculate the covariance of the following pairs of observations of the variables X and Y. (1.5)

(15,14), (20,43), (5,7), (11,15)

- (d) Explain the following terms: (1.5)

1. Mesokurtic
2. Platykurtic
3. Leptokurtic

- (e) What do you understand by the term "Test of Significance"? Explain. (1.5)

- (f) Cards are dealt one by one from a well shuffled pack of playing cards until an ace appears. Find the probability that exactly 'n' cards are dealt before the ace appears. (1.5)

- (g) Write any three properties of Normal Probability Distribution. (1.5)

- (h) Explain the following terms: (1.5)

1. Joint Probability Distribution Function
2. Joint Continuous Density Function
3. Joint Moment Generating Function

- (i) Compute the coefficient of Skewness from the following data: (1.5)

25, 15, 23, 40, 27, 25, 23, 25, 20

- (j) What do you understand by the term 'Attributes'? Explain. (1.5)

PART - B

Q2 (a) State and prove Chebyshev's Inequality. Let X is a variate such that (8)

$$E(x) = 3 \text{ and } E(x^2) = 13$$

then using Chebyshev's Inequality, show that

$$P[-2 < x < 8] \geq \frac{21}{25}$$

(b) The contents of urn I, II and III are as follows: (7)

- One white, 2 black and three red balls
- Two white, one black and one red ball
- Four white, five black and three red balls

One urn is chosen at random and two balls are drawn, they happen to be white and red. What is the probability that they come from

1. Urn I
2. Urn III

Q3 (a) Drive Poisson distribution as a limiting case of Binomial distribution. (8)

(b) The customer accounts of a certain department store have an average balance of Rs. 120 and a standard deviation of Rs. 40. Assuming that the account balances are normally distributed: (7)

1. What proportion of the account is over Rs. 150?
2. What proportion of account is between Rs. 100 and Rs. 150 ?
3. What proportion of account is between Rs. 60 and Rs. 90 ?

[Given : Value of Z at 0.75 is 0.2735

Value of Z at 0.5 is 0.1915

Value of Z at 1.5 is 0.4332]

Q4 (a) Two tetrahedral with sides numbered 1 to 4 are tossed. Let X denotes the number on the downturned face of the first tetrahedron and Y the larger of the downturned numbers. Find (8)

1. the joint density function of X and Y.
2. the marginal density function of X and Y.
3. $\rho(X, Y)$

(b) Calculate Karl Pearson's Coefficient of Correlation from the following data: (7)

| | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|
| X | 24 | 27 | 28 | 28 | 29 | 30 | 32 | 33 | 35 |
| Y | 18 | 20 | 22 | 25 | 22 | 28 | 30 | 27 | 30 |

Q5 (a) Calculate the first four moments about the mean and also the value of β_1 and β_2 from the following data: (8)

| | | | | | |
|---|---|---|---|---|---|
| X | 2 | 3 | 4 | 5 | 6 |
| F | 1 | 3 | 7 | 3 | 1 |

Comment on the skewness and kurtosis of the distribution.

- (b) Calculate mean, median, mode and standard deviation using following data: (7)

| Marks | Number of Students |
|-----------|--------------------|
| Above 0 | 80 |
| Above 10 | 77 |
| Above 20 | 72 |
| Above 30 | 65 |
| Above 40 | 55 |
| Above 50 | 43 |
| Above 60 | 28 |
| Above 70 | 16 |
| Above 80 | 10 |
| Above 90 | 8 |
| Above 100 | 0 |

- Q6 (a) Using least square method, fit a second degree parabola to the following data (8)
 $y = a + bx + cx^2$

| | | | | | |
|---|---|-----|-----|-----|-----|
| X | 0 | 1 | 2 | 3 | 4 |
| Y | 1 | 1.8 | 1.3 | 2.5 | 6.3 |

- (b) In an antimalarial campaign in a certain area Quinine was administered to 1624 persons out of a total population of 6496. The number of fever cases is shown below: (7)

| Treatment | Fever | No Fever | Total |
|------------|-------|----------|-------|
| Quinine | 40 | 1584 | 1624 |
| No Quinine | 440 | 4432 | 4872 |
| Total | 480 | 6016 | 6496 |

Discuss the usefulness of Quinine in checking Malaria.
 (Given Chi-Square at 5% level of significance is 3.84)

- Q7 (a) For the discrete probability distribution (3)

| | | | | | | | | |
|---|---|---|----|----|----|-------|--------|----------|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| f | 0 | k | 2k | 2k | 3k | k^2 | $2k^2$ | $7k^2+1$ |

Determine the value of k and hence calculate mean.

- (b) Differentiate between Gamma distribution and Exponential distribution. (3)

- (c) If X and Y are two random variables having joint density function (3)

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 < x < 2 < y < 4 \\ 0, & \text{Otherwise} \end{cases}$$

- Find 1. $P[X < 1 \cap Y < 3]$
 2. $P[X + Y < 3]$

- (d) From the data given below, calculate the coefficient of Rank Correlation between X and Y. (3)

| | | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|-----|
| X | 78 | 89 | 97 | 69 | 59 | 79 | 68 | 57 |
| Y | 125 | 137 | 156 | 112 | 107 | 136 | 123 | 108 |

- (e) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased. (3)
 (Significant value of Z at 5% level of significance is 1.96)