Sr. No 020303

December 2023

B.Tech. (RAI) – III SEMESTER

Mathematics III (Linear Algebra & Numerical Methods)

Time: 3 Hours **Max. Marks:75**

Instructions: 1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*

2. *Answer any four questions from Part -B in detail.*

3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART –A

Q1 (a) Determine whether or not the vectors $u = (1,1,2), v = (2,3,1), w = (4,5,5) in\ R^3$ are linearly dependent. (1.5)

(b) Determine whether or not $(1,1,1), (1,2,3), (2,-1,1)$ form a basis of R^3. (1.5)

(c) Find the Eigen values and corresponding Eigen vector of matrix $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$. (1.5)

(d) State Cauchy- Schwarz Inequality. (1.5)

(e) If 0.333 is the approximate value of 1/3 , find the absolute and relative error. (1.5)

(f) Given $a = 9.00 \pm 0.05, b = 0.0356 \pm 0.0002, c = 15300 \pm 100, d = 62000 \pm 500$. Find the maximum value of absolute error in a+b+c+d. (1.5)

(g) Develop an algorithm using Newton-Raphson Method , to find the fourth root of positive number N. (1.5)

(h) Prove that $\nabla y_n = h\left(1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \cdots \ldots \ldots\right) y'_n$. (1.5)

(i) Find the missing term in the table : (1.5)

X:	2	3	4	5	6
Y:	45.0	49.2	54.1	.....	67.4

(j) Explain briefly Gauss-Seidel Method for solving system of linear equations. (1.5)

PART –B

Q2 (a) Prove that the set of 2 X 2 matrices $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} a, b, c, d \in R\right\}$ under matrix addition and scalar multiplication is a vector space over R. (8)

(b) Starting with the vector (1,1,0) of R^3 , find a basis of R^3. (7)

Q3 (a) If $A = \begin{bmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{bmatrix}$ (8)

i) Find all eigen values of A.

ii) Find an orthogonal matrix P such that $D = P^{-1}AP$ is diagonal.

(b) Consider a subspace U of R^4 spanned by the vectors (7)

$v_1 = (1,1,1,1), v_2 = (1,1,2,4,), v_3 = (1,2,-4,-3).$

Find an orthogonal and an orthonormal basis of U.

Q4 (a) Using Newton –Raphson Iterative Method , find the root of the equation (8)

$e^x sinx = 1$ correct to 4 decimal places.

(b) Using Regula- Falsi method , find the root of the equation $2x = cosx + 3$ correct to three decimal places. (7)

Q5 (a) From the given table , Calculate cos 25° *and* cos 73° using Gregory-Newton Formula. (8)

X:	10	20	30	40	50	60	70	80
Cos X:	0.9848	0.9397	0.8660	0.7660	0.6428	0.5000	0.3420	0.1737

(b) Interpolate by means of Gauss's Backward formula , the population of a town for the year 1974 , given that :

Year:	1939	1949	1959	1969	1979	1989
Population (in Thousands):	12	15	20	27	39	52

(7)

Q6 (a) Using Newton Divided difference formula , find the missing value from (8)

X:	1	2	4	5	6
Y:	14	15	5	.....	9

(7)

(b) Find a polynomial f(x) by using Lagrange's formula and hence find f(3) for

X:	0	1	2	5
F(x):	2	3	12	147

Q7 (a) Solve by using LU Decomposition Method, (8)

$$10x + y + z = 12; 2x + 10y + z = 13; 2x + 2y + 10z = 14$$

(b) Evaluate the integral $\int_0^6 \frac{e^x}{1+x} dx$ using Simson's 1/3rd rule. (7)

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