6. (a) Show that the following system of equations

$$3x - y + 4z = 3$$
$$x + 2y - 3z = -2$$
$$6x + 5y + pz = -3.$$

has atleast one solution for any real $p$. Find the set of solution when $p = -5$. (8)

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 101$ as a Linear polynomial in A. (7)

7. (a) Reduce the matrix $\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$ to diagonal form. (8)

(b) Find the equation of the tangent plane and equation to the normal to the surface $x^2 - 4y^2 + 3z^2 + 4 = 0$ at the point (3, 2, 1). (7)

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**020101**

December 2023

B.Tech. (RAI) 1st SEMESTER

Mathematics-I (BSC-103RAI)

Time : 3 Hours] [Max. Marks : 75

*Instructions :*

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART–A

1. (a) Evaluate the improper integral $\int_0^\infty e^{-x} \sin x \, dx$, if it exists. (1.5)

(b) Evaluate the integral $\int_0^{\pi/2} \cos^8 x \, dx$. (1.5)

(c) Find the Maclaurin's series expansion of the function $\log_e(1 + x)$, $|x| < 1$. (1.5)

(d) Find the radius of the convergence of the series $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$. (1.5)

(e) Test the convergence of the infinite sequence

$\{\sqrt{n+1}-\sqrt{n}\}$. (1.5)

(f) Find the gradient of the function $g$, where $g(x, y, z) = 3x^2y - y^3z^3$ at the point (1, –2, 1). (1.5)

(g) Find the direction derivative of the function $\varphi$, defined as

$\varphi(x, y, z) = x^2yz + 4xz^2$ at the point (1, –2, 1), in the direction of the vector $2\vec{i} - \vec{j} - \vec{k}$.. (1.5)

(h) State Lagrange's mean value theorem. (1.5)

(i) Find the rank of the matrix $\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. (1.5)

(j) Find the Eigen value of the matrix $\begin{bmatrix} 12 & 5 & 0 \\ 7 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$. (1.5)

## PART–B

2. (a) Show that the evolute of the rectangular hyperbola $xy = c^2$. (8)

(b) Test the convergence of the integral $\int_0^\infty \frac{\cos x}{1+x^2}dx$. (7)



3. (a) Find the shortest and longest distance form the point (1, 2, –1) to the sphere $x^2 + y^2 + z^2 = 24$ using Lagrange's method of constrained maxima and minima. (8)

(b) If $u = (x - y)(y - z)(z - x)$, then prove the following :

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

(ii) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$. (7)

4. (a) Find the Fourier series of the function

$f(x) - x^2, -\pi < x < \pi$. (8)

(b) Check the convergence of the p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}, p > 0$. (7)

5. (a) Using Taylor's theorem, prove that

$\frac{-x^3}{6} + \frac{x^5}{120}$ for $x > 0$. (8)

(b) Find the maxima and minima of the function

$10x^6 - 24x^5 + 15x^4 - 40x^3 + 108$. (7)