013306

December 2023
B.Tech. (Mechanical Engineering)- IIISEMESTER
MATHEMATICS III (BSC 201)

Time: 3 Hours   **Max. Marks:75**

Instructions: 1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*

2. *Answer any four questions from Part -B in detail.*

3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Find a partial differential equation by eliminating a and b from (1.5) $z = ax + by + a^2 + b^2$.

(b) Classify the following partial differential equation: (1.5)

$$\partial^2 z / \partial x^2 + \partial^2 z / \partial y^2.$$

(c) Explain the difference between homogenous and non-homogenous linear partial (1.5) differential equations with example.

(d) What is boundary value problems. (1.5)

(e) If X is binomially distributed with parameters n and p, what is the (1.5) distribution of $Y = n - X$.

(f) Write the limitations of regression. (1.5)

(g) Prove that if one of the regression coefficients is greater than unity then other must (1.5) be less than unity.

(h) For a distribution the mean is 10, variance is 6, γ_1 is 1 and β_2 is 4. Find the first (1.5) four moments about the origin, symbols have their usual meanings.

(i) What is the mean of the $B\left(4, \frac{1}{3}\right)$ binomial distribution? (1.5)

(j) Criticize the following statement: (1.5) "The mean of a binomial distribution is 5 and standard deviation is 3."

PART-B

Q2 (a) Solve $\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2 = \cos mx \, \cos ny$. (7)

(b) Solve $\left(D^2 - DD' + D' - 1\right)z = \cos(x + 2y) + e^y$. (8)

Q3 (a) Using the method of separation of variables, solve $(\partial u/\partial x) = 2(\partial u/\partial t) + u$, where (8)
$u(x,0) = 6e^{-3x}$.

(b) Show that the general solution of wave equation $c^2\left(\partial^2 u/\partial x^2\right) = \partial^2 u/\partial t^2$ is (7)

$u(x,t) = \phi(x + ct) + \psi(x - ct)$, where ϕ and ψ are arbitrary functions.

Q4 (a) Urn A contains 2 white and 2 black balls. Urn B contains 3 white and (7)
2 black balls. One ball is transferred from A to B and then one ball is drawn
out of B. Find the chance that this ball is white. If this ball turns out to be
white, find the probability that the transferred ball was white.

(b) Derive Poisson distribution as the limiting case of Binomial (8)
distribution, where $n \to \infty$, $p \to 0$ and $np = \lambda$ (a finite constant).

Q5 (a) In partially destroyed laboratory record of an analysis of correlation data, the (8)
following results are only legible: variance of x is 9, regression lines: $8x - 10y + 66$
$= 0$ and $40x - 18y = 214$. What are

(i) means of x and y (ii) the S.D. of y (iii) coefficient of correlation between x
and y.

(b) If X is the number scored in a throw of a fair die, show that the Chebyshev's (7)
inequality gives $P[|X - \mu| > 2.5] < 0.47$, while the actual probability is zero.

Q6 (a) If θ is acute angle between two regression lines in case of two variables x and y, (8)
show that $\tan\theta = \dfrac{1-r^2}{r}\left(\dfrac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$ where the symbols have their usual meaning.

Explain the significance of the formula when $r = 0, r = \pm 1$.

(b) By the method of least squares, find the curve $y = a + bx$ that best fits the (7)
following data:

x	0	1	2	3	4
y	1	4	10	17	30

Q7 (a) Establish the relation between moment about the mean and moment about (7) arbitrary point. The first four moments of a distribution about the value 4 of the variate are-1.5, 17, -30 and 108. Calculate the first four moments (i) about mean (ii) about origin (iii) about $x = 2$ and also find β_1 and β_2.

(b) A die is thrown 270 times and the results of these throws are given below: (8)

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not.

(Tabulated value of CHI-SQUARE at 5% level of significance for 5 d.f. is 11.09).