Sr. No- 008305

December 2023

B.Tech.-III SEMESTER(ENC/ECE/EEIOT)

Mathematics-III (BS-301)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Find $L\left[\sin^2 3t\right]$, where L denotes Laplace transform. (1.5)

(b) Explain First shifting property for Laplace transform. (1.5)

(c) Find $L^{-1}\left[\frac{1}{(s+1)^3}\right]$, where L^{-1} denotes inverse Laplace transform. (1.5)

(d) Find the Z-transform of $\frac{1}{(n+1)!}$. (1.5)

(e) State Convolution theorem for Z-transform. (1.5)

(f) Explain Modulation theorem for complex Fourier transform. (1.5)

(g) Find the Fourier sine transform of $\frac{1}{x}$. (1.5)

(h) Define solenoidal vector. (1.5)

(i) Find the divergence of the vector $\vec{V}=(xyz)\hat{i}+(3x^2y)\hat{j}+(xz^2-y^2z)\hat{k}$ at the point (2, -1, 1). (1.5)

(j) State Gauss Divergence theorem. (1.5)

PART -B

Q2 (a) Find the Laplace Transform of the function f(t) defined as $f(t)=|t-1|+|t+1|+|t+2|+|t-2|,\ t\geq 0$. (8)

(b) Evaluate $L\left[\int_0^t e^t \frac{\sin t}{t}\,dt\right]$, where L denotes Laplace transform. (7)

Q3 (a) Solve the following differential equation by using Laplace transform (8)

$$\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=4t+e^{3t}$$, when y(0)=1 and y′(0)=-1.

(b) Find the inverse Laplace transform of $\log\frac{s^2+1}{s(s+1)}$. (7)

Q4 (a) Solve the following difference equation using Z-transform: - (8)

$y_{n+2}+6y_{n+1}+9y_n=2^n$, given that $y_0=y_1=0$.

(b) Use convolution theorem to evaluate $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$. (7)

Q5 (a) Using Parseval's identities, prove that (5)

$$\int_0^\infty \frac{t^2}{(4+t^2)(9+t^2)}dt=\frac{\pi}{10}.$$

(b) Find the Fourier transform of (10)

$$f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$$

Hence evaluate $\int_{-\infty}^{\infty}\frac{\sin as\cos sx}{s}ds$.

Q6 (a) Find the directional derivative of the function $f(x,y,z)=2xy+z^2$ at the point (1, -1, 3) in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$. (7)

(b) Verify Green's theorem in the plane for $\oint_C(3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region defined by x=0, y=0, x + y=1. (8)

Q7 (a) If $\vec{A}=2xz\hat{i}-x\hat{j}+y^2\hat{k}$, evaluate $\iiint_V \vec{A}\,dV$, where V is the region bounded by the surface x=0, y=0, x=2, y=6, z=x², z=4. (7)

(b) If the vector $\vec{F}=(ax^2y+yz)\hat{i}+(xy^2-xz^2)\hat{j}+(2xyz-2x^2y^2)\hat{k}$ is solenoidal, find the value of a. Find also the curl of this solenoidal vector. (8)