Sr. No- 015101

December 2023	
B.Tech(ENC/ECE/EEIOT) 1st SEMESTER	
Mathematics-I(Calculus and Linear Algebra)(BSC-103D)	
Time: 3 Hours	**Max. Marks:75**
Instructions:	1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.* 2. *Answer any four questions from Part -B in detail.* 3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Q.1(a)Find the radius of curvature of the given curve y = 4sinx-sin2x at $x = \pi/2$.

(b)Examine the convergence of the improper integral: $\int_1^\infty \frac{dx}{\sqrt{x}}$.

(c)Define Maclaurin's theorem with Lagrange's form of remainder.

(d)State Lagrange's and Cauchy's mean value theorem.

(e)Expand logsinx in power of x using Taylor's series up to three terms.

(f)State Cauchy's root test to check the convergence of the infinite series.

(g)Prove that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist.

(h)A vector field is given by $\vec{A} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$.Show that the field is irrotational.

(i)Using Rank-Nullity theorem, find the nullity of the given matrix, $A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \end{bmatrix}$

(j)Prove that the given matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal. (1.5*10 = 15)

PART-B

Q.2(a)Find the coordinates of the centre of curvature for any point (x,y) on the parabola $y^2 = 4ax$.Also find the equation of the evolute of the parabola. (7)

(b)Find the surface area of the solid generated by the revolution of the ellipse $x^2+4y^2 =16$ about the major axis. (4)

(c) Using Beta –Gamma function, **evaluate** $\int_0^\infty x^6 e^{-2x} dx$. (4)

Q3(a)Using L'Hospital rule,evaluate $\lim_{x\to 0} \frac{e^x \sin x - x - x^2}{x^3}$. (7)

(b)Examine the applicability of the Rolle's theorem for the given function:

$f(x) = \log(x^2+2) - \log 3$ in [-1,1]. (4)

(c)Find the maximum and minimum value of the given function:

$f(x) = x^5 - 5x^4 + 5x^3 + 10,\ x \in R$ (4)

Q.4(a) **Discuss the convergence of the given infinite series:**

$\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \ldots\ldots\ldots\ldots$ (7)

(b)Find the power series solution of the given differential equation

$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ **about the point x = 0.** (8)

Q.5(a)If $\theta = t^n e^{-\frac{r^2}{4t}}$, find the value of 'n' which will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$. (7)

(b)Using Lagrange's method of multiplier, find the dimension of the rectangular box, open at the top,of maximum capacity whose surface is 432 sq. cm. (8)

Q.6(a)Check the consistency of the system of equations:

$x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ and find the values of x, y, z. (7)

(b)Show that the matrix, $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence find P such that $P^{-1}AP$ is a diagonal matrix. (8)

Q.7(a) Find the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. (7)

(b)Verify Cayley –Hamilton theorem for the given matrix , $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & -2 \end{bmatrix}$,also find A^{-1} for the same. (8)