Sr. No. 007102

December 2023

B.Tech. (Electrical Engineering) - I SEMESTER

BSC-103C

Mathematics-I (Calculus and Differential Equations)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Evaluate $\int_{-\infty}^{0} x \sin x \, dx$ if it exists. (1.5)

(b) Prove that $\Gamma(1) = 1$. (1.5)

(c) Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}, n > 0$. (1.5)

(d) Check the convergence of the series $\sum_{n=1}^{\infty} e^n$. (1.5)

(e) Find the $grad\ \psi$ for the function $\psi = \log(x^2 + y^2 + z^2)$ at the point $(1, 2, 1)$. (1.5)

(f) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dx\, dy\, dz$ (1.5)

(g) Write the expansions of Bessel's functions $J_0(x)$ and $J_1(x)$ (1.5)

(h) Define ordinary point and singular point of a differential equation. (1.5)

(i) Find the singular solution of PDE $z = px + qy + p^2 - q^2$. (1.5)

(j) Form a PDE by eliminating the arbitrary constants λ and μ from $x + \lambda y + \mu = \log_e(az - 1)$. (1.5)

PART -B

Q2 (a) Find the equation of circle of curvature of $\sqrt{x} + \sqrt{y} = 1$ at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$. (5)

(b) Evaluate the area of the surface generated by revolving the portion of the curve $y^2 = 4 + x$ cut off by the straight line $x = 2$ about x –axis. (6)

(c) Find the Taylor's series expansion of $\cos x$ about $x = \frac{\pi}{4}$. (4)

Q3 (a) Determine the values of x for which the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots \infty$ is convergent. (8)

(b) Find the half-range series for $f(x) = (x - 1)^2$ in $0 < x < 1$. Hence, find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$. (7)

Q4 (a) Using the Method of Lagrange Multipliers, find the greatest and smallest values that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (7)

(b) State Green's Theorem. Use it to evaluate $\oint_C e^{-x}(\sin y \ dx + \cos y \, dy)$, where C is rectangular curve with vertices $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$. (8)

Q5 (a) Write an equivalent integral for $\int_0^2 \int_{x^2}^{2x} (4x+2)\,dy\,dx$ with the order of integration reversed by sketching the region of integration. Hence, evaluate the integral. (5)

(b) Find the equation of normal line and tangent plane at the point $(2,-3,18)$ to the surface $x^2 + y^2 - 2xy - x + 3y - z = -4$. (5)

(b) Solve ODE $(y^2 + 2x^2 y)dx + (2x^3 - xy)dy = 0$. (5)

Q6 (a) Find general and singular solutions of ODE $y + px - x^4p^2 = 0$ with $p = \frac{dy}{dx}$. (8)

(b) Solve ODE $(1+2x)^2 \frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$. (7)

Q7 (a) Solve $\frac{d^2y}{dx^2} + y = \frac{\cos x}{\sin^2 x}$ by using method of variation of parameters. (7)

(b) Solve the PDE $x(y-z)\frac{\partial z}{\partial x} + y(z-x)\frac{\partial z}{\partial y} = z(x-y)$. (8)