

December 2023

B.Tech.(EL)- III SEMESTER

Mathematics-III (Probability and Statistics) (ELBS-321)

Time: 3 Hours

Max. Marks:75

- Instructions:**
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
 2. Answer any four questions from Part -B in detail.
 3. Different sub-parts of a question are to be attempted adjacent to each other.

PART -A

- Q1 (a) Define discrete random variable. (1.5)
- (b) Define continuous random variable and its properties. (1.5)
- (c) The joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} 2: & 0 < x < 1, 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$ (1.5)
- Find the marginal density function of X.
- (d) The mean and variance of a binomial distribution are 6 and 4 then find n. (1.5)
- (e) Write the mean and variance of standard normal distribution. (1.5)
- (f) Write the formula for the regression equation of Y on X. (1.5)
- (g) Write the normal equation for the least square curve of the form $y=a+bx+cx^2$. (1.5)
- (h) Write the formula for large sample test for difference of proportion. (1.5)
- (i) What is the chi-square test and write its formula? (1.5)
- (j) Define skewness and kurtosis. (1.5)

PART -B

- Q2 (a) A random variable X has the following probability distribution: (10)

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

- (i) Find k
- (ii) Evaluate $P(X < 6)$
- (b) If X is the number scored in a throw of a fair die, show that the chebychev's inequality gives (5)
- $$P[|X - \mu| > 2.5] < 0.47,$$
- Where μ is the mean of X, while the actual probability is zero.

Q3 (a) A probability curve $y = f(x)$ has a range from 0 to ∞ . If $f(x) = e^{-x}$ then find the mean and variance. (5)

(b) Find the mean, variance and mode of normal distribution. What is the difference between normal distribution and standard normal distribution. (10)

Q4 Two random variables X and Y have the following joint probability density function: $f(x, y) = \begin{cases} 2 - x - y; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0; & \text{otherwise} \end{cases}$ (15)

Find (i) marginal density functions of X and Y.

(ii) Conditional density functions.

Q5 (a) Find the standard deviation and moment generating function of Binomial distribution. (5)

(b) The following table gives the number of aircraft accidents that occurs during the various days of the week. Find whether the accidents are uniformly distributed over the week. (10)

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Number of accidents (f_o)	14	16	8	12	11	9	14
Number of accidents (f_e)	12	12	12	12	12	12	12

Given : the value of chi-square significant at 6 degree of freedom is 12.59 at the 5% level of significance.

Q6 (a) Fit a parabola of second degree to the following data: (10)

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

(b) In a sample of 1000 people in Maharashtra, 540 are rice eaters, and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance. (5)

In a partially destroyed laboratory record of an analysis of correlation data, the following results only are eligible:

Variance of X=9. Regression Equations: $8X - 10Y + 66 = 0$, $40X - 18y = 214$.

What is (i) the mean of X and Y

(ii) the correlation coefficient between X and Y.

Q7 Calculate the correlation coefficient from the following data (10)

X	4.4	6.7	10.5	9.6	12.4	5.5	11.1	8.6	14	10.1	7.2	7.9
Y	586	565	515	532	478	560	493	522	575	490	520	515

(15)