Sr. No.. 008502

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B.Tech. (ECE / EEIOT) 5th SEMESTER

Probability Theory and Stochastic Processes (EC-502)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) State & prove Bayes' theorem. (1.5)

(b) What do you mean by modes of convergence in context of "everywhere" "almost everywhere". (1.5)

(c) What is the complement of a set, and how is it denoted? (1.5)

(d) What is the primary difference between strong and weak laws of large numbers? (1.5)

(e) Define conditional distribution function and explain its properties. (1.5)

(f) Difference between random sequence and random process. (1.5)

(g) Obtain mean value of a sum of N weighted random variables. Also define joint moments about the origin. (1.5)

(h) A fair six-sided die is rolled. Calculate the probability of getting an even number. (1.5)

(i) Given the probability density function (PDF) for a continuous random variable X as $f(x) = 2x$ for $0 \leq x \leq 1$, find the probability that X is between 0.2 and 0.5. (1.5)

(j) If the probability density function (PDF) of a continuous random variable Y is $f(y) = 3y^2$ for $0 \leq y \leq 1$, calculate the expected value $E(Y)$. (1.5)

PART -B

Q2 (a) Suppose the number of phone calls received at a call center follows a Poisson distribution with an average rate of 4 calls per hour. Calculate the probability of receiving exactly 3 calls in a given hour. Determine the mean and variance of the Poisson distribution and discuss their interpretations. (8)

(b) State and prove central limit theorem for equal distributions (7)

Q3 (a) Consider a continuous random variable X with the probability density function given by $f(x) = \frac{3}{2}x^2$ for $0 \leq x \leq 1$. Calculate the cumulative distribution function (CDF) for X and use it to find the probability that X is between 0.4 and 0.8. Determine the variance of X. (8)

(b) Radioactive decay follows an exponential distribution. If a sample of a radioactive substance has a half-life of 10 days, calculate the decay constant λ. Determine the probability that a given atom will decay within 15 days. (7)

Q4 (a) Let X and Y be two random variables with the joint probability distribution given by: $P(X=1, Y=2) = 0.1$, $P(X=2, Y=1) = 0.2$, $P(X=2, Y=2) = 0.3$ (8)

whether X and Y are independent or not. Justify your answer.

(b) What are different types of Random processes. State the condition for wide sense and strict-sense stationary random process. (7)

Q5 (a) Explain in detail the cross correlation functions of input and the output of a LTI systems. (8)

(b) In a manufacturing process, the mean weight of a product is 100 grams with a standard deviation of 5 grams. Use Chebyshev's Inequality to find the probability that a randomly selected product deviates from the mean weight by more than 15 grams. (7)

Q6 (a) Consider a probability space with events A and B. Prove the Law of Total Probability using the definition of conditional probability. Apply Bayes' Theorem to express $P(A|B)$ in terms of $P(B|A)$ and $P(A)$. (8)

(b) (7)

Consider the joint probability density function (PDF) for random variables X and Y:

$$f_{XY}(x,y)= \frac{1}{\pi}e^{-\frac{x^2+y^2}{2}}, \quad -\infty < x, \quad y < \infty$$

Calculate the marginal probability density function $f_X(x)$ for X.

Calculate the marginal probability density function $f_Y(y)$ for Y.

Determine whether X and Y are independent based on the joint distribution.

Q7 (a) Define Limit theorems, Strong and weak laws of large numbers, central limit theorem. Justify your answer with physical interpretation of each statement. (8)

(b) If X and Y are two random variables which are Gaussian. If a random variable z is defined as Z=X+Y. find fz(z). (7)

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