Sr. No 002102

December 2023

B. Tech(CIVIL/ENV/FAE)- 1st SEMESTER

Mathematics-I (Calculus, Multivariable Calculus and Linear Algebra) (BSC-103B)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define Gamma function and write down the value of $\Gamma(\frac{1}{2})$ (1.5)

(b) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$ (1.5)

(c) Explain the physical significance of divergence. (1.5)

(d) Write down the Dirichlet's conditions for the convergence of Fourier series. (1.5)

(e) Check the convergence of the sequence $\left\{\frac{1}{n}\right\}$ and the series $\sum\frac{1}{n}$ (1.5)

(f) Expand $sin\ x$ in powers of x (1.5)

(g) Define Critical point and the Saddle point. (1.5)

(h) Show that $grad\ r = \frac{\vec{r}}{r}$ (1.5)

(i) Explain Taylor theorem with Cauchy's form of remainder. (1.5)

(j) Define Rank of the matrix with appropriate example. (1.5)

PART -B

Q2 (a) Find the coordinates of the centre of curvature for any point (x, y) on the parabola $y^2 = 4ax$. Also find the equation of the evolutes of the parabola. (8)

(b) Evaluate $\int_0^{\pi/2}\sqrt{\sin x}\,dx\ X\ \int_0^{\pi/2}\frac{dx}{\sqrt{\sin x}}$ (7)

Q3 (a) Discuss the convergence of (8)

$$\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Expand $\pi x - x^2$ in a half range sine series in the interval $(0, \pi)$ (7)

Q4 (a) Find the values of a and b such that (8)

$$\lim_{x\to 0} \frac{x(1-a\cos x)+b\sin x}{x^3} = \frac{1}{3}$$

(b) Use mean value theorem to prove that (7)

$$\frac{x}{\sqrt{1-x^2}} \geq \sin^{-1}x \geq 0 \text{ if } 0 \leq x < 1$$

When does the equality hold?

Q5 (a) A rectangular box open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction. (8)

(b) Find out the directional derivative of the function (7)

$f(x,y,z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Q6 (a) Evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\, dydx$ (8)

by changing the order of integration.

(b) Find the volume of solid bounded by the surface (7)

$$(x/a)^{2/3} + (y/b)^{2/3} + (z/c)^{2/3} = 1$$

Q7 (a) Using Gauss Jordon method find out the solution of (8)

$$3x + 3y + 2z = 1,$$
$$x + 2y = 4,$$
$$10y + 3z = -2,$$
$$2x - 3y - z = 5$$

(b) Find out the inverse of the matrix by elementary row operations (7)

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

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