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December 2023

B. Tech. (CE(DS)) 3rd Semester

Mathematics for Data Science (BSC-DS-301)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *Symbols used In this paper have their usual meanings.*

PART–A

1. (a) Determine the number of four-digit decimal numbers if repetition of digits is not allowed. (1.5)

(b) Let $\mathbb{Z}$ be a set of all integers. Then prove that (1.5)

$$\mathbb{R} = \{(a, b): a \in \mathbb{Z}; (a-b) \text{ is an even integer}\}$$

is an equivalence relation on $\mathbb{Z}$.

(c) Define regular expressions. (1.5)

(d) Define a Language L over a finite set of symbols. (1.5)

(e) Define degree of the vertex in a Graph G. (1.5)

(f) If G is a finite group, then prove that for any element $a \in G$, $a^{O(G)} = e$. (1.5)

(g) Explain Normal subgroup with an example. (1.5)

(h) Define Lattice. (1.5)

(i) Explain Regula Falsi Method. (1.5)

(j) Evaluate $\int_0^1 x^2 dx$, using Trapozoidal's tide. (1.5)

PART–B

2. (a) Let n be a positive integer and D_n denotes the set of all divisors of n. Considering the partial order as divisibility on D_n, draw Hass diagram for D_{28}, D_{50} and D_{36}. (7)

(b) Prove that the set of natural numbers with the order relation of divisibility is a partially ordered set. (8)

3. (a) (i) Does there exist a Graph with 8 Vertices with degrees 2,2,3,6,5,7,8,4? Draw such a Graph or explain why it doesn't exist?

(ii) Define spanning tree for a Graph G and also explain directed graph. (8)

(b) If G is a planar graph with e edges, v vertices and r regions, then prove that (7)

$$v - e + r = 2.$$

4. (a) State and prove Langrange's Theorem, (8)

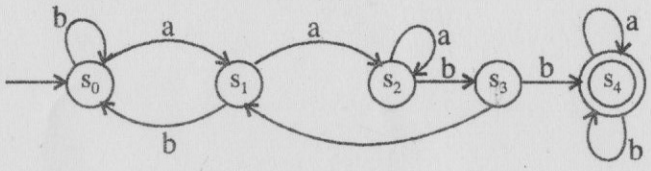
(b) Construct a truth table for the statement of the form : $(p \wedge q) \vee \sim r$, where p, q and r are any propositions and the symbol $\sim r$ denotes the negation of r. (7)

5. (a) Using generating function method, find the total solution of the difference equation $a_n - 4a_{n-1} = 6.\ 4^n$. (8)

(b) Prove that a non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H$ inplies $ab^{-1} \in H$. (7)

6. (a) Find the Language L(G) generated by the grammar G, with variables σ, A, B; T = $\{a, b\}$ and productions P = $\{\sigma \to aB, B \to b, B \to bA$ and $A \to aB\}$. (8)

(b) Define Language determined by a Finite state Automation and Find the language accepted by the automation M shown in the transition diagram below: (7)



7. (a) Find first five iterations of the bisection method to find the smallest positive root of the equation

$$x^3 - 5x + 1 = 0.$$ (8)

(b) Find first four iterations of the Newton-Raphson method to find the smallest positive root of the equation $x^3 - 17 = 0$, with the initial approximation $x_0 = 2$. (7)