6. (a) If $\alpha = (4, 3, 5)$, $\beta = (0, 1, 3)$, $\gamma = (2, 1, 1)$, $\delta = (4, 2, 2)$ in R^3 then prove that

(i) α is a combination of β and γ.

(ii) β is not a linear combination of γ and δ. (8)

(b) Show that the set $S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (0, 1, 0)\}$ spans the vector space R^3 but is not a basis set. (7)

7. (a) Determine the linear mapping $\varphi : R^3 \to R^2$ which maps the basis vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) of R^3 to the vectors (1, 1), (2, 3), (–1, 2) of R^2 respectively. Find out φ(l, 2, 0). (8)

(b) Use Gram-Schmidt process to obtain an orthogonal basis from the basis set {(1, 1, 0), (0, 1, 0), (1, 0, 1)} of R^3 with the standard inner product. (7)

Roll No. .... Total Pages : 4

003101

December 2023

B.Tech. (CE/IT/CE (Hindi Medium)/CE(DS)/CSE(AIML))

1st SEMESTER

Mathematics-I (Calculus and Linear Algebra) (BSC-103E)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$. (1.5)

(b) Define basis and dimensions of a vector space. (1.5)

(c) Write down the relationship between Beta & Gamma function. Find out the value of Beta (2, 3). (1.5)

(d) Define Rank of a matrix. (1.5)

(e) Following vectors are linearly dependent or not (3, 2, 7), (2, 4, 1),(l, –2, 6) (1.5)

(f) Define kernel of a linear mapping. (1.5)

(g) Prove that the matrix A is orthogonal. (1.5)

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$ (1.5)

(h) Write down the radius of curvature of an implicit equation. (1.5)

(i) Find out the sum and product of Eigen values of

$$B = \begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$ (1.5)

(j) Explain Taylor theorem with Cauchy's form of remainder. (1.5)

PART–B

2. (a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^{\frac{5}{2}} x \, dx$. (8)

(b) Find out the coordinates of centre of curvature for any point on the parabola $y^2 = 4ax$, also find out the equation of evaluate of the parabola. (7)

3. (a) Apply Maclaurin's theorem to $f(x) = (1 + x)^4$ to deduce the expression in the power of x. (8)

(b) Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x}}$ (7)

4. (a) Find out the inverse of the matrix C

$$C = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}.$$

Also find out non-singular matrix P & Q such that PCQ = I, where I is identity matrix. Verify that $C^{-1} = QP$. (9)

(b) Find out the rank of the matrix D by reducing it into normal form

$$D = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}.$$ (6)

5. (a) Show that the equations

$$x + 2y - z = 3,$$
$$3x - y + 2z = 1,$$
$$2x - 2y + 3z = 2,$$
$$x - y + z = -1$$

are consistent and solve them. (8)

(b) Show that the transformation is singular

$$y_1 = x_1 - x_2 + x_3,$$
$$y_2 = 3x_1 - x_2 + 2x_3,$$
$$y_3 = 2x_1 - 2x_2 + 3x_3$$

Find out the inverse transformation. (7)