## May 2023

B.Tech (CIVIL) - IV SEMESTER

MATHEMATICS-III (BSC-201)
Time: 3 Hours
Max. Marks:75
Instructions:

1. It is compulsory to answer all the questions ( 1.5 marks each) of Part -A in short
2. Answer any four questions from Part -B in detail.
3. Different sub-parts of a question are to be attempted adjacent to each other.
4. Use of calculator is allowed.

## PART-A

Q1 (a) Solve

$$
\begin{equation*}
\left(D^{2}-2 D D^{\prime}\right) z=0 \tag{1.5}
\end{equation*}
$$

(b) Differentiate between one dimensional and two dimensional Heat equation.
(c) Find particular Integral of

$$
\begin{equation*}
\left(D^{2}+D D^{\prime}\right) z=\sin (x+y) \tag{1.5}
\end{equation*}
$$

(d) Differentiate between Multinomial distribution and Baye's rule.
(e) Cards are dealt one by one from a well shuffled pack of playing cards until an ace appears. Find the probability that exactly " $n$ " cards are dealt before an ace appears.
(f) A random variable X has the following distribution:

| X | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X})$ | 0.1 | K | 0.2 | 2 K | 0.3 | K |

Find the expected value.
(g) Define the following terms:

1. Positive correlation
2. Partial correlation
3. Non-linear correlation
(h) Differentiate between correlation analysis and regression analysis.
(i) Explain Rank Correlation with example.
(j) Explain " Test of significance for a single mean".

## PART-B

Q2 (a) Solve

$$
\begin{equation*}
\left(4 D^{2}-4 D D^{\prime}+D^{\prime 2}\right) z=e^{3 x-2 y}+\operatorname{Sin} x \tag{8}
\end{equation*}
$$

(b) Find the general solution of one dimensional wave equation:

$$
\begin{array}{ll}
\frac{\partial^{2} u}{\partial t^{2}}=\frac{\partial^{2} u}{\partial x^{2}} & ; \quad 0<x<\infty, t>0  \tag{7}\\
u(x, 0)=\operatorname{Cos}\left(\frac{\pi x}{2}\right) & ; \quad 0 \leq x<\infty \\
\frac{\partial u}{\partial t}(x, 0)=0 & ; 0 \leq x<\infty
\end{array}
$$

Q3 (a) Fit a binomial distribution to following data, when tossing 5 coins:

| X | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| f | 2 | 14 | 20 | 34 | 22 | 8 |

(b) A pair of dice is thrown together, find the expected value and variance.

Q4 (a) Fit a Poisson distribution on the following data:

| X | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| f | 192 | 100 | 24 | 3 | 1 |

(b) If the probability density function of a random variable is given by:

$$
f(x)=\left\{\begin{array}{cll}
K\left(1-x^{2}\right) & , & 0 \leq x \leq 1  \tag{7}\\
0 & , & \text { elsewhere }
\end{array}\right.
$$

Find the value of K and hence calculate mean and variance.
Q5 (a) Find the solution of

$$
\frac{\partial^{2} u}{\partial x^{2}}=\frac{1}{c^{2}} \cdot \frac{\partial u}{\partial t} \quad ; \quad 0<x<1, \quad t>0
$$

for which $u(0, t)=u(1, t)=0$ and $u(x, 0)=k \operatorname{Sin} 2 \pi x$
(b) Solve

$$
y z p+z x q=x y
$$

Q6 (a) Calculate Karl Pearson's coefficient of correlation between $\boldsymbol{x}$ and $\boldsymbol{y}$ for the following data:

| x | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

(b) 1. Define Null and Alternative Hypothesis.
2. A random sample of 200 villages was taken from a certain district and the average population per village was found to be 485 with standard deviation of 50 . Another random sample of 200 villages from the same district gave an average population of 510 per village with standard deviation of 40 . Is the difference between the averages of two samples significant? Justify your answer. (Significant value of Z at $5 \%$ level of significance $=1.96$ )

Q7 (a) From the given data obtain two regression equations using the following data:

| X | 2 | 4 | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y | 5 | 7 | 9 | 8 | 11 |

(b) Fit a second degree parabola in the following data:

| X | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y | 1 | 4 | 10 | 17 | 30 |

