Roll No. ....

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323403

May 2023

B.Sc. (Mathematics) SEMESTER IV

Partial Differential Equations (BMH-403A)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Write heat equation in 2-Dimention. (1.5)

(b) Find the partial differential equation from the equation $x + y + z = f(x^2 + y^2 + z^2)$. (1.5)

(c) Find the complementary function of $(x^2D^2 - 4xyDD' + 4y^2D'^2 + 6yD')z = 0$. (1.5)

(d) Check whether the differential equations $p = x^2 - ay$, $q = y^2 - ax$ are compatible and if yes find their common solution. (1.5)

(e) Write the general canonical form for the hyperbolic type of equations. (1.5)

(f) Classify the given partial equation

$(x - y)(xr - xs - ys + yt) = (x + y)(p + q)$. (1.5)

(g) Find the characteristics of $4r + 5s + t + p + q - 2 = 0$. (1.5)

(h) Solve $(D^4 - D'^4)z = 0$. (1.5)

(i) Write the General form of Monge's Subsidiary equations. (1.5)

(j) Solve $(D^3D'^2 + D^2D'^3)z = 0$. (1.5)

PART–B

2. (a) Using Lagrange Subsidiary equations, solve

$(mz - ny)p + (nx - lz)q = ly - mx$. (8)

(b) Using Charpit Method, find the complete integrals of $p = (z + qy)^2$. (7)

3. (a) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + \cos(x + 2y)$. (8)

(b) Solve $x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD')z = \log (y/x) - (1/2)$. (7)

4. (a) Using Monge's Method, Solve

$y^2r + 2xys + x^2t + px + qy = 0$. (8)

(b) Reduce $\frac{\partial^2 z}{\partial x^2} = (1 + y)^2 \left(\frac{\partial^2 z}{\partial y^2}\right)$ to canonical form. (7)

5. Solve the two dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

subject to the conditions $u(0, y, t) = u(1, y, t) = u(x, 0, t) = u(x, 1, t) = 0$, $u(x, y, 0) = $ Asi $\pi x \sin 2\pi y$ and $\left(\frac{\partial u}{\partial t}\right)_{t=o} = 0$. (15)

6. (a) Using Jacobi's Method, Solve

$(x_2 + x_3)(p_2 + p_3)^2 + zp_1 = 0$. (8)

(b) Solve $(D^3 - DD'^2 - D^2 + DD')z = (x + 2)/x^3$. (7)

7. (a) Reduce $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} - 3z = 0$ to canonical form. (8)

(b) Find the solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$ under the conditions $u(0, t) = u(a, t) = 0, \forall t$ and $u(x, 0) = f(x), 0 < x < a$. (7)
