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May 2023

B.Sc. (H) Mathematics-IV SEMESTER

Analytical Geometry (BMH-401A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find the eccentricity of the ellipse $2x^2 + 3y^2 = 1$. (1.5)

(b) Write the equation of the tangent to the curve $ax^2 + 2hxy + by^2 = 0$ at the point (1, 2). (1.5)

(c) Find the polar coordinates of the point whose cartesian coordinates are (0, 1). (1.5)

(d) Find the nature of the conic $\frac{2}{r} = 2 + 2\cos\theta$. (1.5)

(e) What is great circle? (1.5)

(f) Define semi-vertical angle of a cone. (1.5)

(g) Write the equation of a hyperbolic paraboloid. (1.5)

(h) Write the equation of the director circle of the conic $2x^2 + 3y^2 = 1$ at the point $(1, -1)$. (1.5)

(i) What is chord of contact of a conic? (1.5)

(j) Write the general equation for zx plane. (1.5)

PART–B

2. (a) Trace the following conic and reduce it to canonical form (7)

$$4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0.$$

(b) Trace the following conic, reduce it to canonical form and find its eccentricity, and length of major axis

$$x^2 + 12xy - 4y^2 - 6x + 4y + 9 = 0. \quad (8)$$

3. (a) Find the equation of the tangent and normal to the conic $x^2 + 2xy + y^2 - 2x - 1 = 0$ at the point $(0, 1)$. (7)

(b) The normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ meets the parabola again at the point $(at_1^2, 2at_1)$. Prove that $t^2 + tt_1 + 2 = 0$. (8)

4. (a) Find the coordinates of the pole of the straight line $3x + 4y + 1 = 0$ with respect to the circle

$$x^2 + y^2 + 6x + 4y - 3 = 0. \quad (7)$$

(b) The plane $x + 2y + 2z = 6$ meets the co-ordinate axes at A, B, C. Find the equation of the sphere OABC, O being the origin. Also find the radius of the sphere. (8)

5. (a) Find the equation of the cone with vertex at the origin and passes through the curve $ax^2 + by^2 + cz^2 = 1$, $\alpha x^2 + \beta y^2 = 2z$. (7)

(b) Find the equation of the cylinder whose generators are parallel to the line $3x = 3y = z$ and whose guiding curve is $x^2 + y^2 = 1, z = 1$. (8)

6. (a) Find the equations of the tangent plane to the surface $x^2 - 2y^2 + 3z^2 = 2$ which are parallel to the plane $x - 2y + 3z = 0$. (7)

(b) Find the pole of the plane $lx + my + nz = p$ with respect to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (8)

7. (a) If the section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. by the plane $z = 0$ is a rectangular hyperbola, then prove that the locus of the vertex of the cone is $\frac{x^2 + y^2}{a^2 + b^2} + \frac{z^2}{c^2} = 1$. (7)

(b) Show that the section of the ellipsoid

$$9x^2 + 6y^2 + 14z^2 = 3$$

by the plane $x + y + z = 0$ is an ellipse. (8)