(b) Solve the differential equation (8)

$$x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=4\ \text{In}\ x.$$

6. (a) A ball weighing 8 lb falls from rest towards the earth from a great height. As it falls, air resistance acts upon it and we shall assume that this resistance (in pounds) is numerically equal to $2v$, where v is the velocity (in feet per second). Find the velocity and distance fallen at time t seconds. (7)

(b) Solve the differential equation (8)

$$\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}=3x^2+4\sin x-2\cos x.$$

7. (a) Write a short note on radioactive decay. Formulate the differential equation and solve it. (5)

(b) If the half life of a radioactive element is τ, then find the rate constant κ for the radioactive element in terms of τ. (5)

(c) Find the orthogonal trajectories of the family of parabolas (5)

$$y = cx^2.$$

Roll No. ....

Total Pages : 4

323202

May 2023

B.Sc. (MATHEMATICS/MAC)- 2nd SEMESTER

Differential Equations (BMH-202A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find all the solutions of the differential equation (1.5)

$$\frac{dy}{dx}=6x(y-1)^{\frac{2}{3}}.$$

(b) Find the order and degree of the differential equation (1.5)

$$4\frac{d^3y}{dx^3}-\left(\frac{d^2y}{dx^2}\right)^3+5\frac{dy}{dx}+4=0.$$

(c) Examine whether the differential equation is linear or not (1.5)

$$\frac{d^3y}{dx^3}+x\frac{d^2y}{dx^2}+3x^2\frac{dy}{dx}-5y=\sin x.$$

Also, find its order.

(d) Examine whether the differential equation is exact or not (1.5)

$$y^2dx + 2xydy = 0.$$

(e) Show that the solutions e^x, e^{-x} and e^{2x} of (1.5)

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$$

are linearly independent.

(f) Solve the differential equation (1.5)

$$\frac{d^2y}{dx^2} + y = 0.$$

(g) Find the general solution of (1.5)

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0.$$

(h) Write a short note on growth and decay model. Formulate the differential equations. (1.5)

(i) Write a short note on Lotka-Volterra population model. (1.5)

(j) Write the general formula for finding the Picard method of successive approximations for finding a solution of the initial value problem (1.5)

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_0) = y_0.$$

PART–B

2. (a) Solve the differential equation (7)

$$(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0.$$

(b) Solve the initial value problem (8)

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2} \quad y(x_0) = 0, \text{ where } x_0 > 0.$$

3. (a) Find the general and singular solution of the equation (7)

$$p = \log(px - y).$$

(b) Solve (8)

$$\frac{dy}{dx} + y = f(x), \text{ where } f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}$$

and $y(0) = 0$.

4. (a) Solve the initial value problem (7)

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0, \; y(0) = -3, \; y'(0) = -1.$$

(b) Solve the differential equation (8)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10 \sin x.$$

5. (a) Solve the differential equation (7]

$$\frac{d^2y}{dx^2} + y = \tan x.$$