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May 2023
B.Sc. (H) Mathematics - II SEMESTER
and
B.Sc. Mathematics & Computing - II SEMESTER
Real Analysis (BMH-201A)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Show that, for each x in R, there corresponds one and only one real number y such that, (1.5)

$$x + y = y + x = 0$$

(b) For all real numbers x and y, show that (1.5)

$$|x + y| \leq |x| + |y|$$

(c) Find the supremum and infimum of the following: (1.5)

1. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ------\}$
2. $\{\pi + 1, \pi + \frac{1}{2}, \pi + \frac{1}{3}, \pi + \frac{1}{4}, ------\}$
3. $\{-1, \frac{-1}{2}, \frac{-1}{3}, \frac{-1}{4}, ------\}$

(d) Show that, (1.5)

$$\lim \left(\frac{1}{n}\right) = 0$$

(e) Define the following terms with example: (1.5)
1. Increasing Sequence
2. Decreasing Sequence
3. Convergent Sequence

(f) Explain divergence criteria for sequence in R. (1.5)

(g) State Monotone subsequence theorem. (1.5)

(h) Explain the following: (1.5)
1. Limit Superior
2. Limit Inferior
3. Cauchy Sequence

(i) Explain comparison test for series. (1.5)

(j) Explain D'Alembert's ratio test for series. (1.5)

PART-B

Q2 (a) State and prove 'Density Theorem'.

(b) For any real numbers 'a' and 'b', Show that

1. $||a| - |b|| \leq |a - b|$
2. $|a\,b| = |a||b|$

(c) Explain the following terms:

1. Supremum
2. Infimum
3. Completeness property of real numbers

Q3 (a) State and prove Monotone Convergence Theorem.

(b) Let, $X = \langle x_n \rangle, Y = \langle y_n \rangle$ and $Z = \langle z_n \rangle$ are sequences of real numbers such that, $x_n \leq y_n \leq z_n$; *for all* $n \in N$.
And let, $\lim \langle x_n \rangle = \lim \langle z_n \rangle = M$.
Then show that, $Y = \langle y_n \rangle$ is convergent and $\lim \langle y_n \rangle = M$.

(c) Prove that a sequence in real numbers can have at most one limit.

Q4 (a) State and prove Bolzano Weierstrass theorem.

(b) Prove that every convergent sequence of real numbers is a Cauchy sequence.

(c) Prove that, a Cauchy sequence of real numbers is bounded.

Q5 (a) Explain Leibnitz's rule for convergence of an Alternating series and hence test the convergence of the following series:

$$\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} - \text{------} \infty$$

(b) Test the convergence of the following series:

1. $\sum_{n=1}^{\infty} \frac{1}{n^3}$
2. $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$
3. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
4. $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$
5. $\sum_{n=2}^{\infty} \frac{1}{n \log n}$

Q6 (a) What do you understand by bounded sequence? Show that a convergent sequence of real numbers is bounded. (4)

(b) Let $X = x_n$ and $Y = y_n$ be sequences of real numbers that converges to x and y respectively. Prove that, the sequence $X + Y = < x_n + y_n >$ converges to $x + y$. (4)

(c) State and prove Archimedean Property. (7)

Q7 (a) State and prove " Cauchy's convergence criterion". (7)

(b) Explain the following test for series with example: (8)

a. Cauchy's n^{th} root test
b. Cauchy's Integral test
c. Limit Comparison test
d. Caushy's criterion for series.
