Roll No. ....

323406

May 2023

B.Sc. (H) MATHEMATICS (Re-Appear) - IV SEMESTER

RING THEORY AND LINEAR ALGEBRA (BMH-402)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define the following terms: (1.5)
1. Ring
2. Subring
3. Field

(b) Show that, the characteristic of an integral domain is either zero or a prime number. (1.5)

(c) Explain Ideal test. (1.5)

(d) Let R be a ring. Show that the identity map I on R is a ring homomorphism. (1.5)

(e) Define the following terms: (1.5)
1. Ring Homomorphism
2. Ring Isomorphism
3. Kernel of Ring Homomorphism

(f) Differentiate between Linearly Independent and Linearly Dependent vectors. (1.5)

(g) Show that, $\boldsymbol{Symm(M_n(F))}$ forms a subspace of $\boldsymbol{M_n(F)}$. (1.5)

(h) Show that the union of two subspaces of a vector space V need not form a subspace of V. (1.5)

(i) Differentiate between Nullity of a linear transformation and Rank of a linear transformation. (1.5)

(j) Define the following terms: (1.5)
1. Linear Transformation
2. Identity transformation
3. Zero Transformation

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PART -B

Q2 (a) Prove that, every field is an integral domain. Is converse true? Justify your answer. (8)

(b) Define the term Ideal and hence prove that the union of two ideals is an deal if and only if one of them is contained in the other one. (7)

Q3 (a) State and prove First Isomorphism theorem for rings. (8)

(b) Prove the followings: (7)

1. Let, $\emptyset : R \rightarrow S$, be a ring homomorphism. If $\emptyset$ is an isomorphism then show that, $\emptyset^{-1}$ is also an isomorphism.
2. Let, $\emptyset : R \rightarrow S$, be a ring homomorphism. If R is commutative then show that, $\emptyset(R)$ is also commutative.

Q4 (a) Show that R^n forms a vector space over the field of real numbers. (8)

(b) Show that, (7)

1. $\{1, \sqrt{3}\}$ is linearly independent in R over the field of rational numbers.
2. $\{1, \sqrt{3}, \sqrt{5}\}$ is linearly independent in R over the field of rational numbers.

Q5 (a) State and prove Dimension Theorem. (8)

(b) Suppose that V and U be finite dimensional vector spaces over the field F. Then show that V is isomorphic to U if and only if dim (V) = dim (U). (7)

Q6 (a) Let R be a commutative ring with unity and let M be a proper Ideal of R, Then show that $\frac{R}{M}$ is a field if and only if M is Maximal Ideal of R. (8)

(b) State and prove Third Isomorphism theorem for rings. (7)

Q7 (a) Let, $\beta = \{v_1, v_2, v_3, ---, v_n\}$ be a subset of a vector space V over F. Then show that β forms a basis for V if and only if every element of V can be expressed uniquely as a linear combination of elements $v_1, v_2, v_3, ---, v_n$. (8)

(b) Let V and U be vector spaces over a field F and let $T, S : V \rightarrow U$ be a linear transformations. Then show that, (7)

1. $aT + S$ is linear, for all $a \in F$
2. $L(V, U)$ forms a vector space over F.