Sr. No. 323407

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B.Sc. (H) Mathematics Re-Appear - IV SEMESTER

Partial Differential Equations (BMH-403)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Define the following terms: (1.5)
1. Complete Integral
2. General Integral
3. Singular Integral

(b) Find the order and degree of the following partial differential equation: (1.5)

$$\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{1/2}$$

(c) Give an example of non-linear partial differential equation of order one. (1.5)

(d) Differentiate between Linear and Non-Linear homogenous partial differential equations. (1.5)

(e) Solve (1.5)

$$r = a^2 t$$

(f) Differentiate between Hyperbolic, Elliptical and Parabolic PDE. (1.5)

(g) Define Monge's Method. (1.5)

(h) Define the following terms: (1.5)
1. Heat equation (two dimension)
2. Heat equation (one dimension)
3. Laplace equation

(i) Differentiate between characteristics equations and characteristics curves of the second order partial differential equation. (1.5)

(j) Define wave equation in Cartesian coordinates. (1.5)

PART -B

Q2 (a) Solve: (8)

$$(x^2 + 2y^2)\,p - x\,y\,q = x\,z$$

(b) Find the complete integral of (7)

$$p^2 - y^2 q = y^2 - x^2$$

Using charpit's method.

Q3 (a) Solve the following: (8)

1. $(D^3 - 6D^2D' + 11DD'^2 - 6D'^3)z = 0$

2. $\frac{\partial^3 z}{\partial x^3} - 7\left(\frac{\partial^3 z}{\partial x\, y^2}\right) + 6\left(\frac{\partial^3 z}{\partial y^3}\right) = 0$

(b) Solve (7)

$$(D^2 + DD' + D' - 1)z = Sin(x + 2y)$$

Q4 (a) Classify the following partial differential equations: (8)

1. $\frac{\partial^2 u}{\partial x^2} + 4\left(\frac{\partial^2 u}{\partial x\, \partial y}\right) + 4\left(\frac{\partial^2 u}{\partial y^2}\right) = 0$

2. $\frac{\partial^2 u}{\partial x^2} + 4\left(\frac{\partial^2 u}{\partial x\, \partial y}\right) + 3\left(\frac{\partial^2 u}{\partial y^2}\right) = 2$

3. $xy\,r - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2)$

4. $u_{xx} + u_{yy} = u_{zz}$

(b) Reduce the following equation into canonical form: (7)

$\frac{\partial^2 z}{\partial x^2} + 2\left(\frac{\partial^2 z}{\partial x\, \partial y}\right) + \left(\frac{\partial^2 z}{\partial y^2}\right) = 0$

Q5 Find the solution of the wave equation (15)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

Using the method of separation of variables.

Q6 (a) Show that the equations (8)

$$x\,p - y\,q = x \quad and \quad x^2 p + q = x\,z$$

Are compatible and hence find their solution.

(b) Solve (7)

$$x^2 r - y^2 t + px - qy = logx$$

Q7 (a) Solve (8)

$$r - t\cos^2 x + p\tan x = 0$$

Using Monge's Method

(b) Explain the followings: (7)

1. Principle of Superposition.
2. Method of separation of variables.

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