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May 2023

B.Sc. (Chemistry) IV SEMESTER

Ordinary and Partial Differential Equations (OMTH-402)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Solve $(y \cos x + 1)dx + \sin x\, dy = 0$. (1.5)

(b) Solve $(p - q)(z - px - qy) = 1$. (1.5)

(c) Solve the Differential Equation

$$\frac{d^2y}{dx^2} + (a + b)\frac{dy}{dx} + aby = 0. \quad (1.5)$$

(d) If $u = \sin^{-1} x/y + \tan^{-1} y/x$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. (1.5)

(e) If $x^n + y^n = a^n$, find $\frac{d^2y}{dx^2}$. (1.5)

(f) Solve the differential equation, $y = px + p^3$ and find the singular solution. (1.5)

(g) Write the differential equation of a LCR circuit with EMF E. (1.5)

(h) Solve the differential equation

$(D^3 - 3D^2 + 4D - 2)y = 0.$ (1.5)

(i) Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0.$ (1.5)

(j) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0.$ (1.5)

PART–B

2. (a) Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}.$ (8)

(b) Solve the following Differential Equation

$(xy^3 + y)\,dx + 2(x^2 y^2 + x + y^4)dy = 0.$ (7)

3. (a) Using Charpit's Method, Solve $z^2(p^2z^2 + q^2) = 1.$ (8)

(b) Solve $z^2(p^2 + q^2) = x^2 + y^2.$ (7)

4. (a) Solve $z^2(p^2 + q^2 + 1) = a^2.$ (8)

(b) Find the dimension of a rectangular box, open at the top, of maximum capacity whose surface area is 432 sq. cm. (7)

5. (a) Prove that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube. (8)

(b) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u \text{ and}$$

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = \sin 4u - \sin 2u.$$ (7)

6. (a) Solve the differential equation $(D^4 + 16D^2)y = x^2 + 5.$ (8)

(b) Solve the differential equation

$(D^3 + 2D^2 + D)y = x^2e^{2x} + \sin^2 x.$ (7)

7. (a) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}.$$ (8)

(b) A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original?