Sr. No 321205

May 2023

B.Sc.(Physics) - II SEMESTER (Re-Appear)

Linear Algebra (OMTH-201)

Time: 3 Hours

Max. Marks:75

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *Any other specific instructions*

PART -A

Q1 (a) Prove that the only Eigen values that an Idempotent Matrix can possess are either zero or one. (1.5)

(b) State Triangle's inequality. (1.5)

(c) Find the Rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. (1.5)

(d) If A and B are orthogonal matrices prove that AB is also orthogonal. (1.5)

(e) Define orthogonality and orthonormality of Inner Product Spaces. (1.5)

(f) Define Rank and Nullity of a Linear Transformation. (1.5)

(g) By Giving suitable example show that union of two subspaces may not be a subspace of a vector space. (1.5)

(h) Define the following terms: (1.5)

1. Linear combination
2. Spanning set

(i) Show that, (1.5)

$$|(\alpha, \beta)| \leq ||\alpha|| \; ||\beta||$$

(j) Check whether the linear Transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (x + z, x - z, y)$ is invertible or not. (1.5)

PART -B

Q2 (a) Find the Eigen vectors and their corresponding eigen vectors of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.Also find eigen values of A^{-1}. (8)

(b) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ to its normal form and hence find its Rank. (7)

Q3 (a) Find a matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to its diagonal form. (8)

Hence calculate A^3.

(b) For what values of parameters λ, μ do the system of equations (7)

$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have i) no solution ii) unique solution iii) more than one solution?

Q4 (a) State and Prove Rank –Nullity Theorem. (8)

(b) Check whether the set (1,1,2), (1,2,5) and (5,3,4) is a basis of R^3 or not? (7)

Q5 (a) State and prove Cauchy Schwatz Inequality. (8)

(b) Show that $T: R^4 \to R^4$ defined by $T(x, y, z, t) = (2x, 3y, 0,0)$is a linear Transformation. Find its rank and nullity. (7)

Q6 (a) If $T: R^3 \to R^3$ is a linear Transformation defined by (8)

$T(x, y, z) = (x + y, x - z, y)$ show that T is invertible and hence calculate T^{-1}.

(b) If V and W are finite dimensional vector spaces over a same field F then show that V is isomorphic to W iff Dim V=Dim W (7)

Q7 If T is a linear operator defined by (15)

$T: R^3 \to R^3$,

$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$ then find the matrix of T with respect to the ordered basis $B = \alpha_1 = (1,0,1), \alpha_2 = (-1,2,1), \alpha_3 = (2,1,1)$