May 2023, Reappear
B.Sc. (H) Physics, Semester-IV
Mathematical Physics-III (BPH-401)

Time: 3 Hours Max. Marks:75

Instructions:
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
2. Answer any four questions from Part -B in detail.
3. Different sub-parts of a question are to be attempted adjacent to each other.

PART -A

Q1 (a) Show that the function u = 3x-2xy is a harmonic function. (1.5)

(b) Express the complex number $z = \dfrac{1+7i}{(2-i)^2}$ into polar form. (1.5)

(c) Define an analytic function $f(z)$. What are the necessary and sufficient condition for $f(z)$ to be analytic in a region R. (1.5)

(d) Evaluate $\int_C \dfrac{dz}{z(z+\pi i)}\,dz$ where C is a circle $|z+3i|=1$, by Cauchy's integral formula. (1.5)

(e) Find the roots of $z^4 + 1 = 0$. (1.5)

(f) Discuss the singularity of the function $f(z) = \sin\left(\dfrac{1}{z-a}\right)$ at z = a. (1.5)

(g) Explain the change of scale property of Laplace transform. (1.5)

(h) If $F\{f(t)\} = F(s)$ then show that $F\{f(x-a)\} = e^{isx}F(s)$ (1.5)

(i) Evaluate the Fourier Transform of Dirac-delta function. (1.5)

(j) Find the Laplace transform of (1+ sin 2t). (1.5)

PART -B

Q2 (a) Let f(z)= u(x,y)+iv(x,y) be an analytic function. If u = 3x-2xy, then find the imaginary part of f(z). (7)

(b) Evaluate $\int_c \dfrac{z^2 - 2z}{(z+1)^2(z+2i)}\,dz$; using residue theorem where c is the circle $|z|=2$. (8)

Q3 (a) Evaluate $\int_c \dfrac{z^2 +1}{(z+1)(z+2)}\,dz$; using Cauchy's integral formula where c is the circle $|z|=\dfrac{3}{2}$ (7)

(b) By contour integration, prove that $\displaystyle\int_0^\infty \dfrac{\sin mx}{x}\,dx = \dfrac{\pi}{2}$. (8)

Q4 (a) Show that $\log z = (z-1) - \dfrac{(z-1)^2}{2} + \dfrac{(z-1)^3}{3} + \ldots\ldots$ (5)

(b) Expand $\cos z$ in a Taylor series about z = π/4. (5)

(c) (5)

Expand $f(z) = \dfrac{z}{z^2 - 3z + 2}$ in a Laurent series in the region $|z-1| > 1$

Q5 (a) Find the Fourier transform of the function $f(x) = \begin{cases} 1 \ for \ |x| < a \\ 0 \ for \ |x| > a \end{cases}$ (7)

(b) State and Prove convolution theorem on Fourier Transform. (8)

Q6 (a) Find the Fourier Cousine transform of $f(x) = e^{-ax}$. (7)

(b) Obtain the Laplace transform of $t^2 e^t.\sin 4t$. (8)

Q7 (a) Evaluate the integral $\displaystyle\int_0^\infty \dfrac{e^{-t} \sin t}{t} dt$ and $\displaystyle\int_0^\infty \dfrac{\sin t}{t} dt$ using Laplace transformation. (7)

(b) Evaluate $L^{-1}\left\{\dfrac{3s+1}{(s-1)(s^2+1)}\right\}$ using partial fraction method. (8)

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