6. (a) State and prove Convolution theorem on Fourier transform. (7)

(b) Solve the following differential equation using Laplace transform

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t$$

where $y(0) = 1, \left(\frac{dy}{dt}\right)_{t=0} = 0, \left(\frac{d^2y}{dt^2}\right)_{t=0} = -2.$ (8)

7. (a) Compute $L\left\{\int_0^t e^{-2t} t \sin^3 t \, dt\right\}$. (5)

(b) Evaluate $\int_0^\infty \frac{e^{-t} - e^{-4t}}{t} dt$. (5)

(c) Find the inverse Laplace transform of

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)}.$$ (5)

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May, 2023

B.Sc. (Physics) - IV Semester

MATHEMATICAL PHYSICS-III (BPH-401A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) What is Cauchy-Riemann for analyticity of a complex function? (1.5)

(b) Show that $\log \frac{x+iy}{x-iy} = 2i \tan^{-1}\frac{y}{x}$. (1.5)

(c) Find the pole of $f(z) = \frac{\sin(z-a)}{(z-a)^4}$. (1.5)

(d) If $\alpha - i\beta = \frac{1}{a-ib}$, prove that $(\alpha^2 + \beta^2)(a^2 + b^2) = 1$. (1.5)

(e) Explain simply and multiply connected region along with examples. (1.5)

(f) What is Dirac-delta function? Evaluate

$$\int_{-\infty}^{\infty} \sin 2t\, \delta\left(t-\frac{\pi}{4}\right) dt.$$ (1.5)

(g) Prove that $F\left\{\int_a^x f(x)\,dx\right\} = \frac{F(s)}{(-is)}$. (1.5)

(h) Compute $L^{-1}\left\{\frac{1}{2s(s-3)}\right\}$. (1.5)

(i) Show that $L\{u(t-a)\} = \frac{e^{-as}}{s}$, where $u(t-a)$ is unit step function. (1.5)

(j) Prove that $L[f'(t)] = sL[f(t)] - f(0)$. (1.5)

PART–B

2\. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that (8)

(i) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3/2$.

(ii) $\cos^2(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha) = \sin(\alpha+\beta) + \sin(\beta+\gamma) + \sin(\gamma+\alpha) = 0$.

(b) If ω is a cube root of unity, prove that $(1-\omega)^6 = -27$. (7)

3\. (a) Explain in brief three types of singularities : removable, essential and branch points, along with examples of each. (8)

(b) Using Contour integration method, prove the following integrals : (7)

(i) $\int_0^{\infty} \frac{x^{a-1}}{1+x}\,dx = \frac{\pi}{\sin \pi\alpha}$ $(0 < a < 1)$.

(ii) $\int_0^{\infty} \frac{x^{a-1}}{1-x}\,dx = \pi \cot \pi a$.

4\. (a) Find Taylor expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = 1$. (5)

(b) Evaluate using Cauchy's Integral Formula $\int_C \frac{dz}{z^2-1}$, where C is the circle $x^2 + y^2 = 4$. (5)

(c) State and prove Cauchy's Inequality condition. (5)

5\. (a) Express $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ 0 & \text{for } \quad x > \pi \end{cases}$ as a Fourier sine integral and hence evaluated $\int_0^{\infty} \frac{1-\cos\pi\lambda}{\lambda} \sin\lambda x\, d\lambda$. (7)

(b) Find Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate Fourier sine transform of $xe^{-a^2x^2}$. (8)