January, 2023

B. Tech. (CE) Re-Appear MATHEMATICS-III SEMESTER

MATHEMATICS-III (HAS-203)

Time: 3 Hours **Max. Marks:60**

Instructions:
1. *It is compulsory to answer all the questions (2 marks each) of Part –A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Explain Fourier transform. (2)

(b) State Convolution theorem for Fourier integrals. (2)

(c) Find Fourier series representing $f(x) = x,\ 0 < x < 2\pi.$ (2)

(d) Write DIRICHLET'S CONDITIONS for Fourier series. (2)

(e) Evaluate the following integral : (2)

$$\oint_C \frac{z-3}{z^3+z}dz,\ C : |z| = 2.$$

(f) State necessary condition for analytic function. (2)

(g) Explain singularity of a complex function with example. (2)

(h) If A and B are independent, $P[A] = \frac{1}{3}$ and $P[\overline{B}] = \frac{1}{4}$, find $P[A \cup B]$. (2)

(i) If $f_1(x)$ and $f_2(x)$ are probability density functions, show that $(\theta+1)\ f_1(x) - \theta\ f_2(x),\ 0 < \theta < 1,$ is probability density function. (2)

(j) Define optimal feasible solution and basic feasible solution. (2)

PART -B

Q2 Expand the function $f(x) = x \sin x,$ as Fourier series in the interval $-\pi < x < \pi$ (10)

And deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \ldots = \frac{1}{4}(\pi - 2).$

Q3 (a) Obtain half range cosine series for $f(x) = x^2$ in the interval $0 \le x \le \pi.$ (6)

(b) Show that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin (4)

Q4 Determine the analytic function $f(z)=u+iv$, if $u-v=\dfrac{\cos x+\sin x-e^{-y}}{2(\cos x-\cosh y)}$ and $f\left(\dfrac{\pi}{2}\right)=0$. (10)

Q5 (a) Find radius of convergence of the series $f(z)=\sum_{n=1}^{\infty}(n+a^n)z^n$, where a is a complex constant. (5)

(b) Evaluate the integral $I=\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta)d\theta$. (5)

Q6 (a) An urn contain 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the probability that it is a white ball? (5)

(b) Is the sum of two independent binomial variates a binomial variate ? If not, what are the conditions under which it is so? (5)

Q7 Apply dual simplex method, solve the following LPP (10)

Min. $Z=5x_1+6x_2$

Subject to the constraints

$x_1+x_2\geq 2,\ 4x_1+x_2\geq 4,$ and $x_1,x_2\geq 0.$

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