

Jan. 2023

BCA- I SEMESTER

Mathematics (BCA-17-103)

Time: 3 Hours

Max. Marks:75

- Instructions:**
1. It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.
 2. Answer any four questions from Part -B in detail.
 3. Different sub-parts of a question are to be attempted adjacent to each other.

PART -A

- Q1 (a) If $A = \{1,3,5,7\}$, $B = \{2,4,6,8\}$ and $C = \{0,4,5\}$, then find $(A \cup B) \cap C$. (1.5)
- (b) Write the given set in Roaster form $B = \{x: 4x - 3 < 6, x \in N\}$. (1.5)
- (c) If $A = \{2, 3, 4\}$ and $B = \{3,6,8\}$, then find R where R is the relation 'x divides y' from set A to set B. Also find R^{-1} . (1.5)
- (d) If $f: R \rightarrow R$ is defined by $f(x) = 3x^2 - 8x + 1$, then find $f[f(x)]$. (1.5)
- (e) Evaluate $\lim_{x \rightarrow 1} \frac{(x^3 - 1)}{(x - 1)}$. (1.5)
- (f) Find all co-factors of the given matrix (1.5)
- $$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \\ 7 & 0 & -1 \end{bmatrix}.$$
- (g) Discuss the continuity of function $f(x) = x^3 + x^2 - 1$. (1.5)
- (h) Differentiate the given(w.r.t. x): $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$. (1.5)
- (i) Differentiate the given(w.r.t. x): $y = (x)^{\frac{5}{2}} \cdot (x)^{\frac{-2}{3}}$. (1.5)
- (j) Evaluate the given integral: $\int \sin^2(x/2) dx$. (1.5)

PART -B

- Q2 (a) In a Class of 1000 students, 625 students pass in mathematics and 525 pass in english. Then find how many students pass in mathematics only and how many students pass in English only? (10)
- (b) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then show that $A^3 - 3A - 2I = 0$ and hence find A^{-1} . (5)

Q3 (a) Prove that f is a bijective function and hence find its inverse where $f : R \rightarrow R$ (5)
is defined as $f(x) = x^3 + 9$.

(b) Determine whether the relation R in the set $A = \{4, 5, 6, 7\}$ and defined as (10)
 $R = \{(4, 5), (5, 4), (7, 6), (6, 7)\}$ is reflexive, symmetric, transitive or antisymmetric.

Q4 Solve the following system of equations using crammer's rule: (15)

$$2x + 8y + 5z = 5$$

$$x + y + z = -2$$

$$x + 2y - z = 2$$

Q5 (a) By using first principle, find the derivative of $f(x) = (x^2 - 6) / 3x$. (5)

(b) If $y = x + \sqrt{x^2 - 1}$, then prove that $(y - x) \frac{dy}{dx} - y = 0$. (10)

Q6 (a) Evaluate $\int \tan^{-1}(\sec x + \tan x) dx$. (10)

(b) Evaluate $\int \left(\frac{x^4 + 1}{x^2 + 1} \right) dx$. (5)

Q7 If (15)

$$f(x) = \begin{cases} \frac{|x-2|}{2-x}, & x \neq 2 \\ -1, & x = 2 \end{cases}, \text{ then show that } f \text{ is discontinuous at } x=2. \text{ Also write the}$$

type of discontinuity.
