(b) Find the radius of curvature of the curve $xy^2 = b^3 - x^3$ at $(b, 0)$. (7)

7. (a) Find the maximum and minimum values of $f(x, y) = x^2 - xy + y^2 - 2x + y$. (8)

(b) If $y = \sin(m \sin^{-1} x)$. Then prove that $(1 - x^2)y_2 - xy_1 + m^2y = 0$ and also deduce that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$. (7)

Roll No. .... Total Pages : 4

321107

January, 2023

B.Sc. (Physics/Chemistry) Re-Appear Ist Semester

CALCULUS (OMTH-101)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find all the second-order partial derivative of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. (1.5)

(b) Calculate $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x^2 - 9}$. (1.5)

(c) Find the radius of curvature of the curve $x = e^t \cos t$, $y = e^t \sin t$, $z = \frac{t^2}{2}$ at $t = 0$. (1.5)

(d) State Euler's theorem for Homogeneous functions. (1.5)

(e) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$. (1.5)

(f) Change the order of integration in $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dydx$. (1.5)

(g) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz \, dxdydz$. (1.5)

(h) Evaluate $\int_0^1 x^{11} (1 - x)^5 dx$. (1.5)

(i) What are Taylor's and Maclaurin's Series for one variable. (1.5)

(j) Find all the stationary points of the function $f(x, y) = 3x^2 - 2xy + y^2 - 8y$. (1.5)

PART–B

2. (a) Find the value of the constant k, if possible, that will make the function $f(x)$ continuous everywhere.

(i) $f(x) = \begin{cases} 7x - 2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$

(ii) $f(x) = \begin{cases} 2x + k, & x > 2 \\ kx^2, & x \leq 2 \end{cases}$ (7)

(b) Using Maclaurin's series expand $\tan x$ up to the term containing x^5. (8)

3. (a) If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2} \tan u.$$ (7)

(b) Find the shortest and longest distance from the point (1, 2, –1) to the sphere $x^2 + y^2 + z^2 = 24$, using Lagrange's method of constrained maxima and minima. (8)

4. The area bounded by $y^2 = 4x$ and the line $x = 4$ is revolved about the line $x = 4$. Find the volume of the solid of revolution. (15)

5. (a) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes (i.e. $x = 0$, $y = 0$, $z = 0$). (8)

(b) Using beta and gamma function evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} \, d\theta$. (7)

6. (a) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ about the major axis. (8)