(b) Evaluate $\iint_R y\,dxdy$, where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (7½+7½)

7. (a) Find the volume of the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$.

(b) Prove that $\int_0^1 x^m(1-x^n)^p = \dfrac{1}{n}\dfrac{\Gamma(p+1)\Gamma\left(\frac{m+1}{n}\right)}{\Gamma\left(p+1+\frac{m+1}{n}\right)}$ use it to find $\int_0^1 x^7(1-x^4)^9 dx$ in terms of gamma functions.

(7½+7½)

Roll No. ....

321103

January, 2023

B.Sc. (Physics) Ist Semester

CALCULUS

Paper–OMTH-101A

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Evaluate $\lim_{x\to 0}\dfrac{1-\cos x}{\sin^2 x}$.

(b) Write nth derivative of $\dfrac{1}{2x+1}$.

(c) Find asymptotes parallel to co-ordinate axes of $y = e^x$.

(d) Find $\dfrac{\partial u}{\partial x}$ if $u = \log(x^2+y^2)$.

(e) If $r = \sqrt{x^2 + y^2}$ and $\theta = y/x$, then evaluate $\frac{\partial(r, \theta)}{\partial(x, y)}$.

(f) Write the surface of solid generated by the revolution about the x – axis, of the area bounded by the curve $x = f(t)$, $y = \phi(t)$, the x – axis and the ordinates at the points, $t = a$, $t = b$.

(g) Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$.

(h) Evaluate $\int_0^1 x^7 (1-x)^6 \, dx$.

(i) Change into polar co-ordinates (r, θ), the integral

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy\,dx.$$

(j) Define Gamma function and write value of $\lceil 3/2$.

(10×1.5=15)

PART–B

2. (a) If $y = \left[\log(x + \sqrt{1+x^2})\right]^2$, prove that $(1 + x^2)y_{n+2} + (2n + 1)x\,y_{n+1} + n^2 y_n = 0$.

(b) Expand by Maclaurin's theorem $\frac{e^x}{1+e^x}$ as far as x^3.

(7½+7½)

3. (a) Find all the asymptotes of the curve $x(y - x)^2 - x(y - x) = 2$.

(b) Find the radius of curvature for the curve $x^{2/3} + y^{2/3} = a^{2/3}$ at (x, y). (7½+7½)

4. (a) State Euler's theorem on homogeneous functions. If

$$u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right), \text{ then find value of } x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}.$$

(b) If $u = x^2y^3$, $x = \log t$, $y = e^t$ then find $\frac{du}{dt}$. (7½+7½)

5. (a) Expand $e^x \sin y$ in power of x and y as far as terms of third degree.

(b) Find the value of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x–axis. (7½+7½)

6. (a) Find the surface of the solid generated by the revolution of the ellipse $x^2 + 4y^2 = 16$ about its major –axis.