Sr. no. 753103

January 2023
M. Sc. Mathematics - I Semester
Ordinary Differential Equations (MATH21-703)

Time: 3 Hours **Max. Marks: 75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

Q1 (a) Solve the differential equation $y^2dx + 2xydy = 0$. (1.5)

(b) Check if the solutions e^x, e^{-x}, e^{2x} of a differential equation are linearly independent or not. (1.5)

(c) Find the singular points of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2+2)y = 0$. (1.5)

(d) Give an example of a system of two linear differential equations with two unknown functions in normal form. (1.5)

(e) Find the adjoint equation of $(2t+1)\frac{d^2x}{dt^2} + t^3\frac{dx}{dt} + x = 0$. (1.5)

(f) Determine the nature of the critical point (0,0) of the system

$$\frac{dx}{dt} = 2x + 4y, \quad \frac{dy}{dt} = -2x + 6y$$

(g) State Lipschitz condition w.r.t y. (1.5)

(h) Explain negative semi-definite function with an example. (1.5)

(i) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}(x-2)^n$. (1.5)

(j) What is the degree of the DE $(\frac{d^3y}{dx^3})^{2/3} + (\frac{d^3y}{dx^3})^{3/2} = 0$? (1.5)

PART-B

Q2 (a) Find the power series solution in powers of $(x-1)$ of the initial value problem (7)

$$xy'' + y' + 2y = 0, \ y(1) = 1, \ y'(1) = 2$$

(b) Solve the following DE in series of $x = 0$ (8)

$$2x^2y'' - xy' + (1-x^2)y = 0$$

Q3 (a) Find the third approximation of the solution of $\frac{dy}{dx} = 2 - \frac{y}{x}$ by Picard's method, where $y(1) = 2$. (7)

(b) State and prove Cauchy-Peano existence theorem. (8)

Q4 (a) Find the general solution of the following linear system (7)

$$\begin{aligned} \frac{dx}{dt} &= 3x + 2y \\ \frac{dy}{dt} &= -5x + y \end{aligned}$$

(b) State and prove Sturm separation theorem. (8)

Q5 (a) Transform the following equation into an equivalent self-adjoint equation (7)

$$f(t)\frac{d^2x}{dt^2} + g(t)x = 0$$

(b) Solve the following linear system by using operator method (8)

$$\begin{aligned} 2\frac{dx}{dt} + \frac{dy}{dt} + x + 5y &= 4t \\ \frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y &= 2 \end{aligned}$$

Q6 (a) Define saddle point of a linear system and explain with figure. (7)

(b) Find the characteristic values and characteristic functions of the following Sturm-Liouville problem (8)

$$\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = 0,\ y(L) = 0,\ L > 0$$

Q7 (a) Find the derivative of the function $E(x, y) = x^2 \tan y$ w.r.t the system (7)

$$\begin{aligned} \frac{dx}{dt} &= -x + y^2 \\ \frac{dy}{dt} &= -y + x^2 \end{aligned}$$

(b) Determine the type and stability of the critical point (0, 0) of the nonlinear autonomous system (8)

$$\begin{aligned} \frac{dx}{dt} &= x + x^2 - 3xy \\ \frac{dy}{dt} &= -2x + y + 3y^2 \end{aligned}$$