Sr. No. 753106

January 2023

M.Sc. Mathematics (Re-Appear) - I SEMESTER

Real Analysis (MATH 17-101)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) State Dirichlet's Test. (1.5)

(b) State Weierstrass Approximation Theorem. (1.5)

(c) Define Point wise convergence. (1.5)

(d) State Darboux's condition of integrability. (1.5)

(e) Find the radius of convergence of (1.5)

$$\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$$

(f) Define Power series. (1.5)

(g) Show that: (1.5)

$$\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2} = 0$$

(h) Differentiate between measurable sets and measurable function. (1.5)

(i) Define the term Borel Set. (1.5)

(j) Distinguish between partition and refinement of partition (1.5)

PART -B

Q2 (a) Prove that, a sequence of functions $\{ f_n \}$ defined on [a, b] converges uniformly on [a, b] if and only if for every $\varepsilon > 0$ and for all x in [a, b], there exists an integer N such that, $| f_{n+p}(x) - f_n(x) | < \varepsilon$, for every $n \geq N$ and $p \geq 1$. (8)

(b) State and prove Abel's Test. (7)

Q3 (a) Prove that every continuous function is integrable. (6)

(b) Prove that, a bounded function f is integrable on [a, b] if and only if for every $\varepsilon > 0$ there exists a partition P of [a, b] such that, (9)

$$U(P, f) - L(P, f) < \varepsilon$$

Q4 (a) If a power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = x_0$, then prove that it is absolutely convergent for every $x = x_1$, when $|x_1| < |x_0|$. (6)

(b) State and prove Taylor's theorem. (9)

Q5 (a) Let $< f_n >$ be a sequence of non-negative measurable functions, then show that, (8)

$$\int \liminf f_n \, dx \leq \liminf \int f_n dx$$

(b) For any sequence of sets $\{E_n\}_n$ (7)

$$m^*\left(\coprod_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} m^*(E_n)$$

Q6 (a) Prove that the outer measure of an interval is equals to its length. (9)

(b) If f_1 and f_2 are two bounded and integrable functions on [a, b], then show that their product $f_1 \cdot f_2$ is also bounded and integrable on [a, b]. (6)

Q7 (a) Write a short note on " Uniform convergence on an Interval ". (6)

(b) If, (9)

$$f(x, y, z) = \frac{a^2x^2 + b^2y^3 + c^2z^2}{x^2y^2z^2}$$

Where $ax^2 + by^2 + cz^2 = 1$ and a, b and c are positive. Then show that, the minimum value of f(x, y, z) is given by:

$$x^2 = \frac{u}{2a(u+a)}, \quad y^2 = \frac{u}{2b(u+b)}, \quad z^2 = \frac{u}{2c(u+c)}$$

Where u is the positive root of the equation:

$$u^2 - (bc + ca + ab)u - 2abc = 0$$

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