January 2023
MCA-1st SEMESTER
Mathematical Foundations of Computer Science
(MCA-17-101)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Write the regular expression corresponding to following language : (1.5)

   L = {00, 001, 0011, 00111 ....}

   (b) What is difference between NFA and DFA? (1.5)

   (c) Let $f(x) = x^2 + 7$ and $g(x) = 2x - 3$. Define fog(x) and gof(x). (1.5)

   (d) Define tautology with the help of a suitable example. (1.5)

   (e) Let p and q be proposition variables. Construct the truth table for the compound proposition $(\sim p \lor \sim q) \land (p \land q)$. (1.5)

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7. (a) Convert the following Mealy machine to equivalent Moore machine. (5)

Current state	Input Symbol			
	A		b	
	Next State	Output	Next State	Output
→q_0	q_0	1	q_2	1
q_1	q_2	0	q_1	1
q_2	q_1	1	q_1	0

   (b) Write short note on Chomsky Hierarchy of grammars. (5)

   (c) Make a DFA corresponding to the following regular expression: 1*(10)*1*. (5)

(f) What is LUB and GLB in a lattice? (1.5)

(g) What is a multigraph? (1.5)

(h) Define cut point in a graph with the help of a suitable example. (1.5)

(i) Write truth table for $p \leftrightarrow q$ and $p \to q$. (1.5)

(j) What is a Eulerian circuit? What is the necessary condition for its existence in a graph? (1.5)

PART–B

2. (a) Let there be a relation R over the set A = {1, 2, 3, 4} such that aRb iff $a + b \geq 4$. Write R, make digraph for R and write matrix representation of the relation. (5)

(b) What is an equivalence relation? How it creates the partition of the set over which it is defined? Explain with the help of a suitable example. (5)

(c) What a partial order relation? Let there be a set A = {1, 2, 3, 4} and a relation R is defined on A such that aRb iff $a \geq b$. Show that R is a partial order relation. (5)

3. (a) Write a short note on permutation group. (5)

(b) State and prove Lagrange's theorem. (5)

(c) Find DNF for $\sim(P \lor Q) \leftrightarrow (P \land Q)$ (5)

4. (a) Let A = {1, 2, 3} be a set and P(A) be its power set. Show that $(P(A), \subseteq)$ is a poset. Make the Hasse diagram and check if it forms a lattice or not. (7)

(b) Explain the following with the help of a suitable example: Complemented lattice, Distributed lattice. (8)

5. (a) Describe the necessary and sufficient conditions for two graphs to be isomorphic. Illustrate with example.

(b) Find the shortest path between source A and destination C in the following graph. (5)
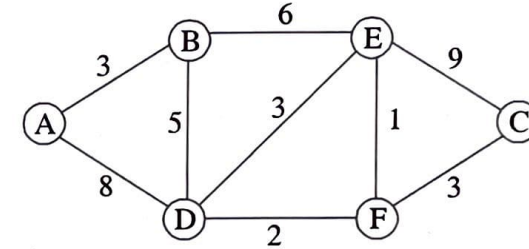
(c) Draw a complete graph of 5 vertices (K5). Use Euler's formula to prove that the graph is not planar. (5)

6. (a) Differentiate between homomorphism and isomorphism. (5)

(b) Find the Hamiltonian cycle in the following graph. (5)
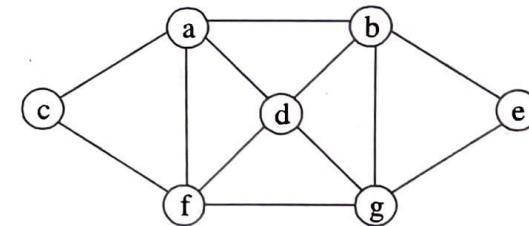
(c) Express the following sentences using suitable predicates and quantifiers. (5)

(i) All that glitters is not gold.

(ii) There exists something which glitters and is not gold.