Sr. N0.323102

January 2023

B.Sc.Mathematics (Hons.) , B.Sc Mathematics & Computing I SEMESTER

Algebra (BMH-102A)

Time: 3 Hours **Max. Marks:75**

Instructions:

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *Any other specific instructions*

PART -A

Q1 (a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & -9 \\ 3 & 6 & 4 \end{bmatrix}$. (1.5)

(b) Show that every square matrix can be expressed in one and only one way as sum of a symmetric and a skew symmetric matrix. (1.5)

(c) Prove that the set of vectors u=(1,3,2),v=(1,-7,-8),w=(2,1,-1) is linearly dependent. (1.5)

(d) Show that the diagonal elements of Hermitian matrix are all real. (1.5)

(e) Express the matrix $\begin{bmatrix} 5-3i & 2i & 3+5i \\ 6 & 7i & 1-4i \\ -2+7i & 4i & 7 \end{bmatrix}$ as sum of Hermitian and a skew hermitian matrix. (1.5)

(f) Find the condition that the roots of the equation $x^3 - ax^2 + p = 0$ may be A.P. (1.5)

(g) Use method of synthetic division to express $f(x) = x^4 + 2x^3 + 6x^2 - 8x + 4$ as a polynomial in powers of (x-3). (1.5)

(h) Remove the Fractional coefficients from the equation $x^4 + \frac{1}{2}x^3 - \frac{5}{2}x^2 + \frac{2}{3}x - 1 = 0$ (1.5)

(i) Apply Descarte's rule of siqns to prove that all the roots of the equation (1.5)

$$x^6 - 3x^2 - x + 1 = 0$$

(j) If A and B are orthogonal Matrices . Prove that AB is also orthogonal. (1.5)

PART -B

Q2 (a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (8)

(b) For what values of parameters λ and μ do the system of equations (7)

$$x + y + z = 6; x + 2y + 3z = 10; x + 2y + \lambda z = \mu$$

Have i) no solution ii) unique solution iii) more than one solution

Q3 (a) Verify Cayley Hamilton Theorem for the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ (8)

(b) Find the matrix P which Transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form . (7)

Hence calculate A^4.

Q4 (a) Show that the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence find P such that $P^{-1}AP$ is Diagonal Matrix. (8)

(b) Show that 0 is the characterstic root of a matrix if and only if the matrix is singular. (7)

Q5 (a) Find the common roots of the equations $x^4 + 3x^3 - 5x^2 - 6x - 8 = 0$ and (8)

$$x^4 + x^3 - 9x^2 + 10x - 8 = 0. Hence\ solve\ completely.$$

(b) Solve the equation $x^4 - 8x^3 + 23x^2 - 28x + 12 = 0$, given that the difference of two of its roots is equal to the difference of other two. (7)

Q6 (a) Solve the equation $x^4 - 9x^2 + 4x + 12 = 0$ given that it has multiple root. (8)

(b) Diminish the roots of $2x^5 - x.^3 + 10x - 8 = 0$ by 5 (7)

Q7 (a) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ $by\ Carden's Method.$ (8)

(b) Apply Descarte's Method to solve the equation $x^4 - 3x^2 - 42x - 40 = 0$ (7)

****