Sr. No. 321104

Jan 2023

B.Sc. (H) Physics Semester-IReappear

Mathematical Physics-I (BPH-101)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) State Taylor's theorem for series expansion of an analytic function. (1.5)

(b) Find m so that the vectors $\vec{A} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}; \vec{B} = \hat{\imath} - m\hat{\jmath} + \hat{k}$ and $\vec{C} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ are coplanar. (1.5)

(c) Prove that : $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$ (1.5)

(d) Define Exact differential equation. (1.5)

(e) Solve: $(D^2 - 6D + 25)^2 y = 0$ (1.5)

(f) Find the angle between the surface $x^2+y^2+z^2 = 9$ and $x^2 + y^2 - z = 3$ at $(2,-1,2)$ (1.5)

(g) Solve: $(D^2 + 4)y = \cos 2x$ (1.5)

(h) Define a Solenoidal and an irrotational vector function? (1.5)

(i) Explain Linear Independence and Dependence. (1.5)

(j) Define Dirac-Delta Function and write any of its two properties. (1.5)

PART -B

Q2 (a) Show that the vector field $\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$ is irrotational as well as solenoidal. Find the scalar potential. (7)

(b) Prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$ (8)

Q3 (a) State and verify Green's Theorem in the plane for curve C (7)

$\oint\{(3x^2 - 8y^2)\, dx + (4y - 6xy)dy\}$

Where C is the boundary of the region defined by $y=\sqrt{x}$, and $y= x^2$

(b) Verify Stoke's Theorem for $\vec{F} = (x + y)\hat{\imath} + (2x - z)\hat{\jmath} + (y + z)\hat{k}$ for the surface of a triangular lamina with vertices (2,0,0), (0, 3,0), (0,0,6) (8)

Q4 (a) Starting from the principle, derive an expression for divergence of a vector in orthogonal curvilinear coordinates. (7)

(b) Prove that Spherical polar co-ordinate system is orthogonal (8)

Q5 (a) Solve by method of variation of parameters (7)

$$(D^2 + 2D + 1)y = 4e^{-x}$$

(b) Solve the differential equation (8)

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x \log x$$

Q6 (a) Find the complete solution of the following differential equation (7)

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$$

(b) Solve the following differential equation $(2x \log x - xy)dy + 2ydx = 0$ (8)

Q7 (a) Solve the differential equation: $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ (7)

If the directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at the point $(1,1,1)$ has maximum magnitude 15 in the direction parallel to the line

$$\frac{(x-1)}{2} = \frac{(y-3)}{-2} = \frac{z}{1}$$

(b) Solve the differential equation: $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ (8)

****