Sr. NO. 323105

January 2023

B.Sc. Mathematics(Hons.) - I SEMESTER (Reappear)

Calculus (BMH-101)

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *Any other specific instructions*

PART -A

Q1 (a) Evaluate $\int \sinh^5 x \cosh x \, dx$. (1.5)

(b) Find a,b if $\lim_{x\to 0} \frac{a\sinh x + b\sin x}{x^3} = \frac{5}{3}$. (1.5)

(c) Find the point of inflexion of the curve $y^2 = (x-1)^2(x-2)$. (1.5)

(d) Using reduction formula , evaluate $\int \cos^6 x \, dx$. (1.5)

(e) Sketch the graph of hyperbola $y^2 - x^2 = 1$, showing the vertices ,foci and asymptotes. (1.5)

(f) State theorem of reflection property of Parabolas. (1.5)

(g) Find the maxima or minima of the curve $f(x) = 3x^5 - 5x^3 + 2$. (1.5)

(h) If $\overrightarrow{f(t)} = t\hat{\imath} + (t^2 - 2t)\hat{\jmath} + (3t^2 + 3t^3)\hat{k}$, find $\int_0^1 \overrightarrow{f(t)} dt$. (1.5)

(i) Write the formula of radius of curvature for Cartesian curves. (1.5)

(j) Find the nth derivative of $\log(ax + x^2)$. (1.5)

PART -B

Q2 (a) If $y = (\sin^{-1}x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. Hence, find $(y_n)_0$. (8)

(b) Find all the asymptotes of the following curve $(x-y)^2(x+2y-1) = 3x + y - 7$ (7)

Q3 (a) Deduce reduction formula for $\int sin^m x\, cos^n x\, dx$ and hence evaluate $\int sin^4 x\, cos^2 x\, dx$. (8)

(b) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about x-axis. Find the volume of the solid so generated. (7)

Q4 (a) Describe the graph of the equation $x^2 - y^2 - 4x + 8y - 21 = 0$. (8)

(b) Identify and sketch the curve xy=1 (7)

Q5 (a) Prove that radius of curvature at any point of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is three times the length of the perpendicular from the origin to the tangent at that point (8)

(b) Sketch the curve x = 2 cost, y = 5 sin t, $(0 \le t \le 2\pi)$. (7)

Q6 (a) State and Prove Leibnitz Theorem. (8)

(b) Sketch the graph of $\frac{x^2-x-2}{x-3}$ (7)

Q7 (a) Find the inflection points for the function $f(x) = 3x^5 - 5x^3 + 2$ (8)

(b) Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to an extremity of the latus rectum (7)