(b) Evaluate $\iint_S (2xy\hat{i} + yz^2\hat{j} + xz\hat{k}) \cdot \overrightarrow{ds}$ over the surface of the region bounded by $x = 0$, $y = 0$, $y = 3$, $z = 0$ and $x + 2z = 6$. (5)

7. (a) Prove that the spherical polar co-ordinate system is orthogonal. Hence, express Laplacian operator ∇^2 in spherical polar co-ordinate system, (10)

(b) Express $z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical co-ordinate system. (5)

Roll No. ....

Total Pages : 4

321101

January 2023

B.Sc. - 1st SEMESTER

Mathematical Physics-I (BPH-101A)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Expand the function $f(x) = \frac{1}{x}$ about $a = 1$ using Taylor series. (1.5)

(b) Solve a differential equation $x^2dy + y(x + y)dx = 0$. (1.5)

(c) Prove that the solutions $y_1 = e^{-x} \cos x$ and $y_2 = e^{-x} \sin x$ of a differential equation are linearly independent. (1.5)

(d) Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$. (1.5)

(e) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point (2, –2, 3). (1.5)

(f) Prove that $\vec{\nabla} \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\vec{\nabla} \times \vec{F}) - \vec{F} \cdot (\vec{\nabla} \times \vec{G})$. (1.5)

(g) Show that area of a plane region can be expressed as $A = \frac{1}{2}\int_C (xdy - ydx)$, using Green's Theorem. (1.5)

(h) Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a particle from origin to (1, 1) along a parabola $y^2 = x$. (1.5)

(i) What is Dirac-delta function? Evaluate.

$$\int_{-\infty}^{\infty} \sin 2t\, \delta\left(t - \frac{\pi}{4}\right) dt.$$ (1.5)

(j) Express arc length in cylindrical co-ordinate system co-axial with z-axis. (1.5)

PART–B

2. Solve the following differential equations :

(a) $[1 + \log(xy)]dx + \left[1 + \frac{x}{y}\right] dy = 0.$ (4)

(b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x.$ (5)

(c) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$ using method of variation of parameters. (6)

3. Solve the following differential equations :

(a) $y(x^2y + e^x)dx - e^x dy = 0.$ (4)

(b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = \sinh x + \sin\left(\left(\sqrt{2}\right)x\right).$ (5)

(c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$ using method of variation of parameters. (6)

4. (a) Prove that

$$\vec{p} \times \left[(\vec{a} \times \vec{q}) \times (\vec{b} \times \vec{r})\right] + \vec{q} \times \left[(\vec{a} \times \vec{r}) \times (\vec{b} \times \vec{p})\right] + \vec{r} \times \left[(\vec{a} \times \vec{p}) \times (\vec{b} \times \vec{q})\right] = 0.$$ (5)

(b) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is solenoidal as well as irrotational. Find the scalar potential such that $\vec{F} = \vec{\nabla}\varnothing$. (10)

5. (a) Find the constants 'm' and 'n' such that the surface $mx^2 - 2nyz - (m + 4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2). (5)

(b) State and prove Gauss's Divergence Theorem. (10)

6. (a) Verify Stoke's Theorem for

$$\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\,\hat{j} + (2xz + z^2)\hat{k}$$

over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above the xy-plane. (10)